November 19

SWBAT:

Find the volume by slicing

Volume by slicing

(volume w/ a known cross-section)

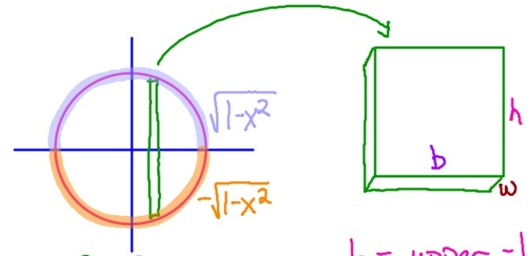
$$V = \int_a^b (\text{Area})\, dx \qquad V = \int_c^d (\text{Area})\, dy$$

a ← x-values c ← y-values

*cross-sections are perpendicular to x-axis

*cross-sections are perpendicular to the y-axis

The base of a solid is bound by the curve $x^2 + y^2 = 1$ the cross-sections perpendicular to the x-axis are squares



$x^2+y^2=1$

$y^2=1-x^2$

$y=\pm\sqrt{1-x^2}$

$h=$ upper $-$ lower

$h=\sqrt{1-x^2}-\left(-\sqrt{1-x^2}\right)$

$h=2\sqrt{1-x^2}$

$b=h=2\sqrt{1-x^2}$

$w=dx$

$\sqrt{1-x^2}=-\sqrt{1-x^2}$

$+\sqrt{1-x^2} \quad +\sqrt{1-x^2}$

$2\sqrt{1-x^2}=0$

$\sqrt{1-x^2}=0$

$1-x^2=0$

$1=x^2$

$\pm 1=x$

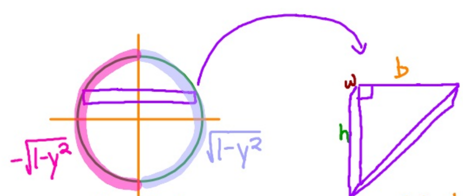
$\text{Area}=bh=\left(2\sqrt{1-x^2}\right)\left(2\sqrt{1-x^2}\right)$

$=\left(2\sqrt{1-x^2}\right)^2=4(1-x^2)$

$\text{Volume}_S=\text{Area}\,dx=4(1-x^2)\,dx$

$\text{Volume}_T=\int_{-1}^{1}4(1-x^2)\,dx=5.333$

The base of a solid is bound by the curve $x^2 + y^2 = 1$ the cross-sections perpendicular to the y-axis are isoceles right triangles with one leg bound by the curve.



$x^2+y^2=1$

$x=\pm\sqrt{1-y^2}$

$b=$ right $-$ left

$b=\sqrt{1-y^2}-\left(-\sqrt{1-y^2}\right)$

$b=2\sqrt{1-y^2}$

$h=b=2\sqrt{1-y^2}$

$w=dy$

$\text{Area}=\frac{1}{2}bh$

$=\frac{1}{2}\left(2\sqrt{1-y^2}\right)\left(2\sqrt{1-y^2}\right)=\frac{1}{2}\left(2\sqrt{1-y^2}\right)^2$

$=\frac{1}{2}\left(4(1-y^2)\right)=2(1-y^2)$

$V_{\text{slice}}=(\text{Area})\,dy=2(1-y^2)\,dy$

$V_{\text{Total}}=\int_{-1}^{1}2(1-y^2)\,dy=2.667$