October 1

SWBAT:
Evaluate definite integrals using the
Fundamental Theorem of Calculus

FTC

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

where $F'(x) = f(x)$

$$\int_1^3 x^2\,dx$$

$$\frac{d}{dx}(?) = x^2$$

$$? = \frac{x^3}{3}$$

$$\int_1^3 x^2\,dx = \frac{x^3}{3}\Big|_1^3 = \frac{3^3}{3} - \frac{1^3}{3} = \frac{27}{3} - \frac{1}{3}$$

$$= \frac{26}{3}$$

$$\int_2^5 x^3\,dx$$

$$\frac{d}{dx}(?) = x^3$$

$$? = \frac{x^4}{4}$$

$$\int_2^5 x^3\,dx = \frac{x^4}{4}\Big|_2^5 = \frac{5^4}{4} - \frac{2^4}{4}$$

$$= \frac{625}{4} - \frac{16}{4} = \frac{609}{4}$$

$$\int_a^b x^n\,dx$$

$$\frac{d}{dx}(?) = x^n$$

$$? = \frac{x^{n+1}}{n+1}$$

$$\int_a^b x^n\,dx = \frac{x^{n+1}}{n+1}\Big|_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$$

$$\int_{-1}^1 2 + x^4 - 3x^2\,dx$$

$$= \int_{-1}^1 2\,dx + \int_{-1}^1 x^4\,dx - \int_{-1}^1 3x^2\,dx$$

$$= 2\int_{-1}^1 1\,dx + \int_{-1}^1 x^4\,dx - 3\int_{-1}^1 x^2\,dx$$

$$\frac{d}{dx}(?) = 1 \qquad \frac{d}{dx}(?) = x^4 \qquad \frac{d}{dx}(?) = x^2$$

$$? = x \qquad ? = \frac{x^5}{5} \qquad ? = \frac{x^3}{3}$$

$$= 2\left(x\Big|_{-1}^1\right) + \frac{x^5}{5}\Big|_{-1}^1 - 3\left(\frac{x^3}{3}\Big|_{-1}^1\right)$$

$$= 2(1-(-1)) + \frac{1^5}{5} - \frac{(-1)^5}{5} - 3\left(\frac{1^3}{3} - \frac{(-1)^3}{3}\right)$$

$$= 2(2) + \frac{1}{5} + \frac{1}{5} - 3\left(\frac{1}{3} + \frac{1}{3}\right)$$

$$= 4 + \frac{2}{5} - 3\left(\frac{2}{3}\right) = 4 + \frac{2}{5} - 2 = 2 + \frac{2}{5}$$

$$\int_{-1}^1 2 + x^4 - 3x^2\,dx$$

$$\frac{d}{dx}(?) = 2 + x^4 - 3x^2$$

$$? = 2x + \frac{x^5}{5} - x^3$$

$$= 2x + \frac{x^5}{5} - x^3\Big|_{-1}^1$$

$$= 2(1) + \frac{1^5}{5} - 1^3 - \left(2(-1) + \frac{(-1)^5}{5} - (-1)^3\right)$$

$$= 2 + \frac{1}{5} - 1 - \left(-2 - \frac{1}{5} + 1\right)$$

$$= 1 + \frac{1}{5} - \left(-1 - \frac{1}{5}\right) = 2 + \frac{2}{5}$$

$$\int_1^9 \frac{4}{x^3}+\sqrt{x}\,dx$$

$$=\int_1^9 4x^{-3}+x^{1/2}\,dx$$

$$\frac{d}{dx}(?)=4x^{-3}+x^{1/2}$$

$$?=4\left(\frac{x^{-2}}{-2}\right)+\frac{x^{3/2}}{3/2}=\frac{4x^{-2}}{-2}+x^{3/2}\left(\frac{2}{3}\right)$$

$$=-2x^{-2}+\frac{2}{3}x^{3/2}$$

$$\int_1^9 \frac{4}{x^3}+\sqrt{x}\,dx=-2x^{-2}+\frac{2}{3}x^{3/2}\Big|_1^9$$

$$=-2(9)^{-2}+\frac{2}{3}(9)^{3/2}-\left(-2(1)^{-2}+\frac{2}{3}(1)^{3/2}\right)$$

$$=-2\left(\frac{1}{9^2}\right)+\frac{2}{3}(\sqrt{9})^3-\left(-2\left(\frac{1}{1^2}\right)+\frac{2}{3}(\sqrt{1})^3\right)$$

$$=\frac{-2}{81}+\frac{2}{3}(27)-\left(-2+\frac{2}{3}\right)$$

$$=\frac{-2}{81}+18-\left(-\frac{4}{3}\right)=\frac{-2}{81}+18+\frac{4}{3}$$

$$\int_0^\pi \sin x-e^x\,dx$$

$$\frac{d}{dx}(?)=\sin x-e^x$$

$$?=-\cos x-e^x$$

$$\int_0^\pi \sin x-e^x\,dx=-\cos x-e^x\Big|_0^\pi$$

$$=-\cos(\pi)-e^\pi-\left(-\cos(0)-e^0\right)$$

$$=-(-1)-e^\pi-(-1-1)$$

$$=1-e^\pi-(-2)=3-e^\pi$$