October 12

SWBAT:
Apply definite integrals to real world applications

2002 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

$$\int_9^{17} E(t)\,dt = \int_9^{17} \frac{15600}{t^2-24t+160}\,dt = 6004$$

\$104,041

$$\int_9^{17} E(t)\,dt = \underline{6004} \qquad \int_9^{23} E(t)\,dt = 7275$$

$7275 - 6004 = 1271$ $\qquad \int_{17}^{23} E(t)\,dt = 1271$

$6004(15) = 90,060$

$1271(11) = 13,981$ $\;>\; 104,041$

GO ON TO THE NEXT PAGE.

when given a rate, how do you find the amount?

Take the integral (anti-derivative)

$$\int_a^b \text{rate } dt = \text{Total Amount from } t=a \text{ to } t=b$$

(rate = derivative)

2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

$\int_9^t E(x)\,dx$ = total # that enter the park

$\int_9^t L(t)\,dt$ = total # that leave the park

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

H(17) is the number of people in the park at t=17

$$H(t) = \int_9^t E(x) - L(x)\,dx$$

$$\frac{d}{dt}\left(\int_9^t E(x) - L(x)\,dx\right) = (E(t) - L(t))(1) - (E(9) - L(9))(0)$$

$$H'(t) = E(t) - L(t)$$

$$H'(17) = -380.281$$

The rate people are entering + leaving is -380.281 ppl/hr