October 15

SWBAT:
Apply definite integrals to real world applications

2002 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

$$H(t) = \int_9^t E(x) - L(x)\,dx$$

$$H'(t) = E(t)(1) - L(t)(1) - (E(9) - L(9))(0)$$

$$H'(t) = (E(t) - L(t))(1)$$

$$0 = E(t) - L(t)$$

$$t = 15.7948$$

max because $H'(t)$ changes from positive to negative

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AP® CALCULUS AB 2002 SCORING GUIDELINES

Question 2

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$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

(a) $\int_9^{17} E(t)\,dt = 6004.270$
6004 people entered the park by 5 pm.

3 : 1 : limits; 1 : integrand; 1 : answer

(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$
The amount collected was \$104,048.
or
$\int_{17}^{23} E(t)\,dt = 1271.283$
1271 people entered the park between 5 pm and 11 pm, so the amount collected was
$\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$

1 : setup

(c) $H'(17) = E(17) - L(17) = -380.281$
There were 3725 people in the park at $t = 17$.
The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.

3 : 1 : value of $H'(17)$; 2 : meanings (1 : meaning of $H(17)$; 1 : meaning of $H'(17)$); $< -1 >$ if no reference to $t = 17$

(d) $H'(t) = E(t) - L(t) = 0$
$t = 15.794$ or 15.795

2 : 1 : $E(t) - L(t) = 0$; 1 : answer

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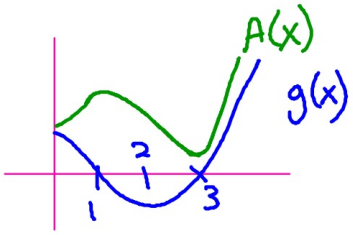
$A(x) = \int_0^x g(t)\,dt$

$A'(x) = g(x)$



$$A(x) = \int_0^x f(t)\,dt$$

$$A'(x) = f(x)$$

$A(x) < 0$