April 15

Given the differential equation:

$$\frac{dy}{dx} = 3x^2y$$

Find the particular solution $y = f(x)$ if $f(1) = 2$

$dy = 3x^2y\,dx$

$\frac{dy}{y} = 3x^2dx$

$\int \frac{1}{y}\,dy = \int 3x^2dx$

$\ln|y| = x^3 + C$

$\ln 2 = 1^3 + C$

$\ln 2 = 1 + C$

$\ln 2 - 1 = C$

$\ln|y| = x^3 + \ln 2 - 1$

$y = e^{x^3+\ln 2-1}$

$e^{x^3}\,e^{\ln 2}\,e^{-1}$

$y = \frac{2e^{x^3}}{e}$

April 15

Students will verbally explain how to Solve problems using calculus

(using the words: derivative, integral, solve, etc...)

	Questions	
Limits	Q5, Q11, Q18, Q21	25
Continuity	Q5, Q9, Q11	33
The Intermediate Value Theorem	Q77	0
The Definition of the Derivative	Q18	0
Differentiabiliy	Q11	0
Computing Derivatives	Q1, Q4, Q7, Q14, Q19, Q20, Q24, Q78	50
Tangent Lines	Q19	100
Critical Points	Q24	100
Increasing & Decreasing Behavior	Q2, Q15, Q17, Q26, Q76, Q80, Q86, Q91	25
Extrema	Q22, Q80, Q82, Q85, Q91	20
Concavity	Q15, Q26, Q76, Q80, Q84, Q87, Q91	29
Implicit Differentiation	Q27	0
Related rates	Q88	100
Riemann Sums	Q8	0
Computing Integrals	Q3, Q12, Q13, Q90	0
Integral as area	Q86	0
Accumulation	Q8, Q15, Q17, Q26, Q81	0
The Fundamental Theorem	Q15, Q17, Q26	0
Area Between Curves	Q10	0
Volume by Slicing	Q92	0
Motion	Q6, Q16, Q28, Q79, Q83, Q89	33
Differential Equations	Q23, Q25	50

	Student Name (Last, First):	
	Enter the Total # of Correct MC questions (out of 45)	12
Question 1 Tabular Data Measuring Temperature (AVG = 3.96, STD DEV = 2.88)	Estimate and interpret the derivative at a point using the average rate of change Part A (2 pts)	0
	Use the FTC to evaluate a definite integral and interpret it in context Part B (2 pts)	0
	Approximate and interpret average value using a Riemann sum Part C (3 pts)	2
	Use the FTC to evaluate a function at a point when given its derivative and an initial value Part D (2 pts)	0
	Total (9 pts)	2
Question 2 Area & Volume (AVG = 3.09, STD DEV = 3.10)	Find the area of a region Part A (3 pts)	0
	Find the volume of a solid by slicing Part B (3 pts)	0
	Use definite integrals to represent two regions of equal area Part C (3 pts)	0
	Total (9 pts)	0

Question 3 Integral-Defined Functions and Interpreting Graphs (AVG = 2.67, STD DEV = 2.58)	Evaluate an integral-defined function using area under the graph of the derivative Part A (2 pts)	0
	Find and evaluate an integral-defined function using the FTC Part B (2 pts)	0
	Identify where an integral-defined function has horizontal tangents and justify if these points are local extrema Part C (3 pts)	0
	Justify where an integral-defined function has points of inflection Part D (2 pts)	0
	Total (9 pts)	0
Question 4 Algebraically Analyzing a Function (AVG = 4.09, STD DEV = 2.61)	Calculate the derivative of a function using the power rule and chain rule Part A (2 pts)	2
	Write the equation of a line tangent to a function at a point Part B (2 pts)	1
	Use the definition of continuity to determine if a piecewise function is continuous Part C (2 pts)	0
	Use u-substitution to evaulate a definite integral Part D (3 pts)	0
	Total (9 pts)	3

Question 5 Differential Equations (AVG = 2.87, STD DEV = 2.24)	Interpret two values of a differential equation in context Part A (2 pts)	0
	Find and interpret the second derivative of a differential equation in context Part B (2 pts)	0
	Use separation of variables to find the particular solution of a differential equation Part C (5 pts)	0
	Total (9 pts)	0
Question 6 Particle Motion (AVG = 3.59, STD DEV = 2.82)	Given velocity, find when a particle is moving to the left Part A (2 pts)	0
	Write an integral that represents the total distance traveled by a particle Part B (1 pts)	0
	Find the acceleration of a particle and use it to identify if its speed is increasing, decreasing or neither at a given time Part C (3 pts)	0
	Find the position of a particle at a given time Part D (3 pts)	0
	Total (9 pts)	0
	Mock Exam Composite Score (108 pts total) 50% MC, 50% FRQ	19
	Mock Exam AP Score 5: 67-108 4: 54-66 3: 41-53 2: 33-40 1: 0-32 AVG = 2.97	1

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Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

$g'(x) = f(x)$

(a) Find the values of $g(2)$ and $g(-2)$.

$\int_1^2 f(t)\,dt$

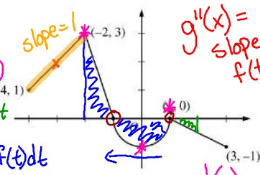
(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

$g'(x) = 0$

$\int_{+1}^{-2} f(t)\,dt$

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.



Graph of $f = g'(x)$

$g''(x) =$ slope of $f(t)$

(a) $g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt$

$= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$

$2 : \begin{cases} 1 : g(2) \\ 1 : g(-2) \end{cases}$

(b) $g'(x) = f(x) \Rightarrow g'(-3) = f(-3) = 2$

$g''(x) = f'(x) \Rightarrow g''(-3) = f'(-3) = 1$

$2 : \begin{cases} 1 : g'(-3) \\ 1 : g''(-3) \end{cases}$

(c) The graph of g has a horizontal tangent line where $g'(x) = f(x) = 0$. This occurs at $x = -1$ and $x = 1$.

$g'(x)$ changes sign from positive to negative at $x = -1$. Therefore, g has a relative maximum at $x = -1$.

$g'(x)$ does not change sign at $x = 1$. Therefore, g has neither a relative maximum nor a relative minimum at $x = 1$.

$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : x = -1 \text{ and } x = 1 \\ 1 : \text{answers with justifications} \end{cases}$

(d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ because $g''(x) = f'(x)$ changes sign at each of these values.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$

because g' has a min or max

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Question 4

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$. $= (25 - x^2)^{1/2}$

$y = \sqrt{25 - (-3)^2}$

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

$4 = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

$4 = \lim_{x \to 3} f(x) = f(-3)$

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\,dx$.

(a) $\boxed{f'(x) = \frac{1}{2}(25 - x^2)^{-1/2}(-2x)} = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$

$2 : f'(x)$

(b) $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$

$f(-3) = \sqrt{25 - 9} = 4$

An equation for the tangent line is $y = 4 + \frac{3}{4}(x + 3)$.

$y - \sqrt{25 - (-3)^2} = \frac{1}{2}(25 - (-3)^2)^{-1/2}(-2(-3))(x - (-3))$

$2 : \begin{cases} 1 : f'(-3) \\ 1 : \text{answer} \end{cases}$

(c) $\lim_{x \to -3^-} g(x) = \lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \sqrt{25 - x^2} = 4$

$\lim_{x \to -3^+} g(x) = \lim_{x \to -3^+} (x + 7) = 4$

Therefore, $\lim_{x \to -3} g(x) = 4$.

$g(-3) = f(-3) = 4$

So, $\lim_{x \to -3} g(x) = g(-3)$.

Therefore, g is continuous at $x = -3$.

$2 : \begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with explanation} \end{cases}$

(d) Let $u = 25 - x^2 \Rightarrow du = -2x\,dx$ $\quad \frac{du}{-2x} = dx$

$\int_0^5 x\sqrt{25 - x^2}\,dx = -\frac{1}{2}\int_{25}^{0} \sqrt{u}\,du$

$= \left[-\frac{1}{2} \cdot \frac{2}{3} u^{3/2}\right]_{u=25}^{u=0}$

$= -\frac{1}{3}(0 - 125) = \frac{125}{3}$

$-\frac{1}{3}(0)^{3/2} - \left(-\frac{1}{3}(25)^{3/2}\right)$

$3 : \begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

Question 5

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

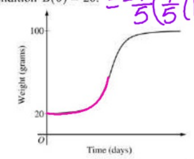
$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

$\frac{1}{5}(0-1)\cdot B' = -\frac{1}{5}B'$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$. $= -\frac{1}{5}\left(\frac{1}{5}(100-B)\right)$

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

(a) $\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(60) = 12$

$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(30) = 6$

Because $\left.\frac{dB}{dt}\right|_{B=40} > \left.\frac{dB}{dt}\right|_{B=70}$, the bird is gaining weight faster when it weighs 40 grams.

$2 : \begin{cases} 1 : \text{uses } \frac{dB}{dt} \\ 1 : \text{answer with reason} \end{cases}$

(b) $\frac{d^2B}{dt^2} = -\frac{1}{5}\frac{dB}{dt} = -\frac{1}{5}\cdot\frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$

Therefore, the graph of B is concave down for $20 \le B < 100$. A portion of the given graph is concave up.

$2 : \begin{cases} 1 : \frac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{cases}$

(c) $\frac{dB}{dt} = \frac{1}{5}(100 - B)$

$\int \frac{1}{100 - B}\, dB = \int \frac{1}{5}\, dt$

$-\ln|100 - B| = \frac{1}{5}t + C$

Because $20 \le B < 100$, $|100 - B| = 100 - B$.

$-\ln(100 - 20) = \frac{1}{5}(0) + C \Rightarrow -\ln(80) = C$

$100 - B = 80e^{-t/5}$

$B(t) = 100 - 80e^{-t/5}, \quad t \ge 0$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } B \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables