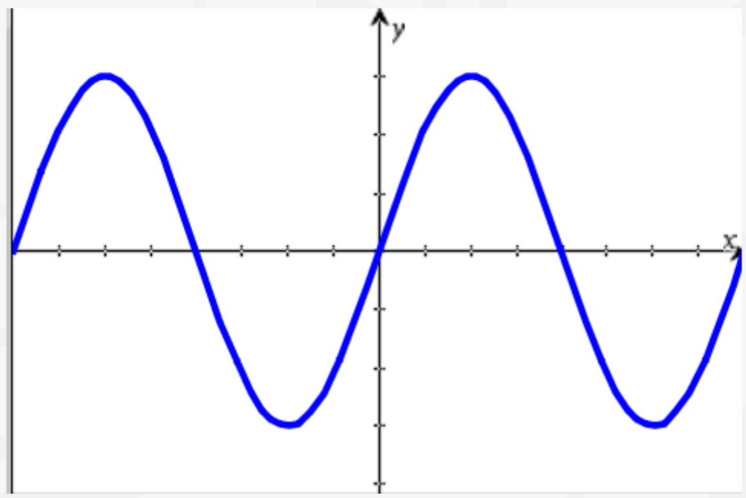
April 17

Given the graph of a function below. Sketch a graph of the first derivative and the second derivative.



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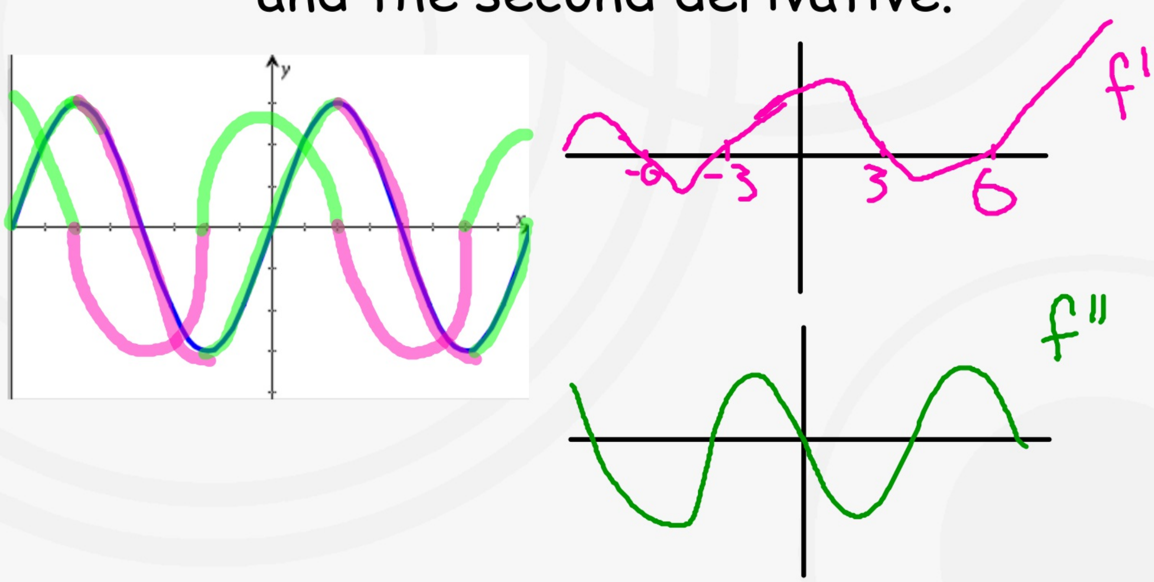

April 17

Students will verbally explain how to

Solve problems using calculus

(using the words:

derivative, integral, solve, etc...)

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Question 5

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

$\frac{1}{5}(0-1)\cdot B' = -\frac{1}{5}B'$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$. $= -\frac{1}{5}\left(\frac{1}{5}(100-B)\right)$

$dB = \frac{1}{5}(100-B)\,dt$

$\frac{dB}{100-B} = \frac{1}{5}\,dt$

$\int \frac{1}{100-B}\,dB = \int \frac{1}{5}\,dt$

$-\ln|100-B| = \frac{1}{5}t + C$

(c) Use separation of variables to find $y = B(t)$, the particular s the differential equation with initial condition $B(0) = 20$.

$-\ln|100-20| = \frac{1}{5}(0) + C$

$-\ln 80 = C$

(a) $\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(60) = 12$

$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(30) = 6$

$u = 100 - B$

$du = -1\,dB$

$-du = dB$

$\int \frac{1}{u}(-du) = -\int \frac{1}{u}\,du$

Because $\left.\frac{dB}{dt}\right|_{B=40} > \left.\frac{dB}{dt}\right|_{B=70}$, the bird is gaining weight faster when it weighs 40 grams.

$2 : \begin{cases} 1 : \text{uses } \frac{dB}{dt} \\ 1 : \text{answer with reason} \end{cases}$

(b) $\frac{d^2B}{dt^2} = -\frac{1}{5}\frac{dB}{dt} = -\frac{1}{5}\cdot\frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$

Therefore, the graph of B is concave down for $20 \le B < 100$. A portion of the given graph is concave up.

$2 : \begin{cases} 1 : \frac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{cases}$

(c) $\frac{dB}{dt} = \frac{1}{5}(100 - B)$

$\int \frac{1}{100 - B}\,dB = \int \frac{1}{5}\,dt$

$-\ln|100 - B| = \frac{1}{5}t + C$

Because $20 \le B < 100$, $|100 - B| = 100 - B$.

$-\ln(100 - 20) = \frac{1}{5}(0) + C \Rightarrow -\ln(80) = C$

$100 - B = 80e^{-t/5}$

$B(t) = 100 - 80e^{-t/5}, \quad t \ge 0$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } B \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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Question 6

$\cos\left(\frac{\pi}{6}t\right)=0$

$\frac{\frac{\pi}{6}t}{\frac{\pi}{6}}=\frac{\frac{\pi}{2}}{\frac{\pi}{6}},\frac{\frac{3\pi}{2}}{\frac{\pi}{6}}$

$v(t)=0$ $t=\frac{\pi}{2}\cdot\frac{6}{\pi}=3$

$t=\frac{3\pi}{2}\cdot\frac{6}{\pi}=9$

moving left when $3<t<9$

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

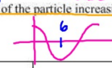
stop		3		9	
sign v	+		−		+
direction	R		L		R

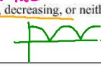
(a) For $0 \le t \le 12$, when is the particle moving to the left? v is neg

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

$\int_0^6 |v(t)|\,dt$ ↳ abs. value

(c) Find the $\overset{v'(t)}{\boxed{\text{acceleration}}}$ of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.





(d) Find the position of the particle at time $t = 4$.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow t = 3, 9$

The particle is moving to the left when $v(t) < 0$.
This occurs when $3 < t < 9$.

$2 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$

(b) $\int_0^6 |v(t)|\,dt$

$\int_0^6 \left|\cos\left(\frac{\pi}{6}t\right)\right|\,dt$

1 : answer

(c) $a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$v(t)=\cos\left(\frac{\pi}{6}t\right)$

$-\sin\left(\frac{\pi}{6}t\right)\left(\frac{\pi}{6}\right)$

$3 : \begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$

$a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$

$\int_0^4 v(t)\,dt = x(t)\Big|_0^4$

$= x(4) - x(0)$

$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$

The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.

(d) $x(4) = -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right)dt$

$= -2 + \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right]_0^4$

$= -2 + \frac{6}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - 0\right]$

$= -2 + \frac{6}{\pi}\cdot\frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi}$

$u=\frac{\pi}{6}t$

$du=\frac{\pi}{6}dt$

$\frac{6}{\pi}du=dt$

$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

$\int \cos(u)\frac{6}{\pi}\,du$

$\frac{6}{\pi}\int \cos(u)\,du$

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