April 18

Find all horizontal asymptotes for the function:

$$f(x) = \frac{\sqrt{4x^2 + 5}}{x - 10}$$

$$\lim_{x \to \infty} \frac{\sqrt{4x^2+5}}{x-10}$$

$$= \lim_{x \to \infty} \frac{\sqrt{4x^2}}{x} = \lim_{x \to \infty} \frac{|2x|}{x} = 2$$

$$\lim_{x \to -\infty} \frac{\sqrt{4x^2+5}}{x-10}$$

$$= \lim_{x \to -\infty} \frac{|2x|}{x} = -2$$

April 18

Students will verbally explain how to

Solve problems using calculus

(using the words:

derivative, integral, solve, etc...)

Test #2:

Multiple Choice

#6, 15, 26, 79, 92

Free Response

#1, 5

Sets 8-13

P. Interpretation of a Derivative as a Rate of Change

What you are finding: As mentioned in section I (Related Rates), a quantity that is given as a rate of change needs to be interpreted as a derivative of some function. Typical problems ask for the value of the function at a given time. These problems can be handled several ways:

a) solving a Differential Equation with initial condition (although DEQ's may not have even been formally mentioned yet)

b) Integral of the rate of change to give accumulated change. This uses the fact that:

$$\boxed{\int_a^b R'(t)\,dt = R(b) - R(a)} \text{ or } R(b) = R(a) + \int_a^b R'(t)\,dt$$

Example 47: A car's gas tank contains 4 gallons. A gas pump can fill the tank at the $\boxed{\text{rate of } \sqrt{9-t}}$ gallons per minute for $0 \le t \le 10$ minutes. How many gallons of gas are in the tank at $t = 5$ minutes?

$t=0$

$f'(t)$

$\int_0^5 f'(t)\,dt = f(5) - f(0)$

$\int_0^5 \sqrt{9-t}\,dt \rightarrow \int_9^4 \sqrt{u}\,(-du) = -\int_9^4 \sqrt{u}\,du$

$u = 9-t$

$du = -dt$

$-du = dt$

$= \int_4^9 u^{1/2}\,du = \left.\frac{u^{3/2}}{3/2}\right|_4^9$

$= \frac{2}{3}\left(9^{3/2} - 4^{3/2}\right) = \frac{2}{3}\left(\sqrt{9}^3 - \sqrt{4}^3\right)$

$= \frac{2}{3}(27-8) = \frac{2}{3}(19) = \frac{38}{3}$

$\int_0^5 f'(t)\,dt = f(5) - f(0)$

$\frac{38}{3} = f(5) - 4$

$f(5) = \frac{38}{3} + 4$

Example 48: (Calc) Frankenstein Electronics makes gold-plated HDMI cables for HD TV's that sell for \$9 a foot. Frankenstein says that the cost of creating an x-foot cable is $\frac{\sqrt{x^3}}{3}$ dollars.

a) Write an expression involving an integral that represents Frankenstein's profit on a cable of length k.

b) Find the maximum profit that Frankenstein could earn on a cable. Justify your answer.

Q. Derivative of Accumulation Function (2nd FTC)

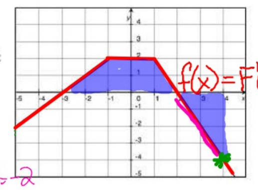
What you are finding: You are looking at problems in the form of $\frac{d}{dx}\int_a^x f(t)\,dt$. This is asking for the rate of change with respect to x of the accumulation function starting at some constant (which is irrelevant) and ending at that variable x. It is important to understand that this expression is a function of x, not the variable t. In fact, the variable t in this expression could be any variable (except x).

How to find it: You are using the 2nd Fundamental Theorem of Calculus that says: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

Occasionally you may have to use the chain rule that says $\boxed{\frac{d}{dx}\int_h^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)} - f(h(x))h'(x)$

Example 51: Let $f(x)$ be defined by the graph to the right whose domain is [-5,5].

Let $F(x) = \int_{-3}^{x} f(t)\,dt$ and $F(-3) = 0$



$f(x) = F'(x)$

a) Put $F(4)$, $F'(4)$ and $F''(4)$ in order from largest to smallest.

$F(4) = \int_{-3}^{4} f(t)dt = 7 - 4 = 3$

$F'(x) = \frac{d}{dx}\int_{-3}^{x} f(t)dt = \underset{f(x)(1) - f(-3)(0)}{f(x)} \rightarrow F'(4) = f(4) = -4$

$F''(x) = \frac{d}{dx}(f(x)) = f'(x) \rightarrow$ slope $\quad F''(4) = f'(4) = -2$

b) Find the equation of the tangent line to F at $x = 4$.

$y - F(4) = F'(4)(x - 4)$

c) Use the results of b) to approximate the value of F at $x = 4.1$. Does this value over-approximate or under-approximate $F(4.1)$? Justify your answer.

d) Find the value of x where F has a maximum. Justify your answer.