April 22

For time $t \geq 0$, the position of a particle traveling along a line is given by a differentiable function s. If s is increasing for $0 \leq t < 2$ and s is decreasing for $t > 2$, what is the total distance the particle travels for $0 \leq t \leq 5$?

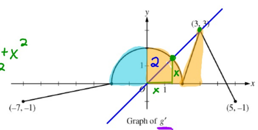
$$\int_0^5 |s'(t)|\,dt$$

$$\int_0^2 s'(t)\,dt - \int_2^5 s'(t)\,dt$$

April 21

Students will verbally explain how to solve problems using calculus (using the words: integral, derivative, slope, etc...)

$2^2 = x^2 + x^2$
$4 = 2x^2$
$2 = x^2$
$\sqrt{2} = x$



Graph of g'

$\int_0^3 g'(x)\,dx = g(3) - g(0)$
$\frac{1}{4}\pi(2)^2 + \frac{1(3)}{2}$
$\pi + \frac{3}{2} = g(3) - 5$
$\pi + \frac{3}{2} + 5 = g(3)$
$\pi + 6.5 = g(3)$

$\int_{-2}^0 g'(x)\,dx = g(0) - g(-2)$
$\frac{1}{4}\pi(2)^2 = 5 - g(-2)$
$\pi = 5 - g(-2)$
$g(-2) = 5 - \pi$

5. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning. $x = 0, 2, 3$ because the slope of $g'(x)$ changes sign $\rightarrow g'(x)$ has a max or min

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

$h'(x) = g'(x) - x$
$0 = g'(x) - x$
$x = g'(x)$
$x = 3, \sqrt{2}$

CP		$\sqrt{2}$	3
sign h'	+	−	−
behav. h	inc	dec	dec

max at $x = \sqrt{2}$ because $h'(x)$ changes from pos to neg
neither max or min at $x = 3$ because the sign of $h'(x)$ does not change

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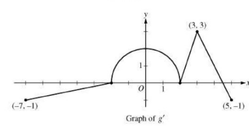
Question 5



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(a) $g(3) = 5 + \int_0^3 g'(x)\,dx = 5 + \frac{\pi \cdot 2^2}{4} + \frac{3}{2} = \frac{13}{2} + \pi$

$g(-2) = 5 + \int_0^{-2} g'(x)\,dx = 5 - \pi$

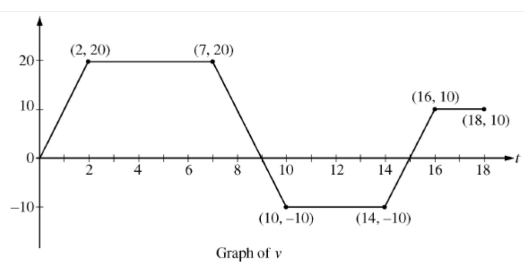
3 : { 1 : uses $g(0) = 5$; 1 : $g(3)$; 1 : $g(-2)$ }

(b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.

2 : { 1 : identifies $x = 0, 2, 3$; 1 : explanation }

(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$
On the interval $-2 \leq x \leq 2$, $g'(x) = \sqrt{4 - x^2}$.
On this interval, $g'(x) = x$ when $x = \sqrt{2}$.
The only other solution to $g'(x) = x$ is $x = 3$.
$h'(x) = g'(x) - x > 0$ for $0 \leq x < \sqrt{2}$
$h'(x) = g'(x) - x \leq 0$ for $\sqrt{2} < x \leq 5$
Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.

4 : { 1 : $h'(x)$; 1 : identifies $x = \sqrt{2}, 3$; 1 : answer for $\sqrt{2}$ with analysis; 1 : answer for 3 with analysis }



Graph of v

4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \leq t \leq 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \leq t \leq 18$ is the squirrel farthest from building A? How far from building A is the squirrel at that time?

(c) Find the total distance the squirrel travels during the time interval $0 \leq t \leq 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

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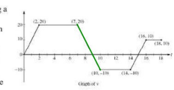
Question 4

A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B. For $0 \leq t \leq 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.



(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \leq t \leq 18$ is the squirrel farthest from building A? How far from building A is the squirrel at this time?

(c) Find the total distance the squirrel travels during the time interval $0 \leq t \leq 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

(a) The squirrel changes direction whenever its velocity changes sign. This occurs at $t = 9$ and $t = 15$.

2 : { 1 : t-values; 1 : explanation }

(b) Velocity is 0 at $t = 0$, $t = 9$, and $t = 15$.

t	position at time t
0	0
9	$\frac{9+5}{2} \cdot 20 = 140$
15	$140 - \frac{6+4}{2} \cdot 10 = 90$
18	$90 + \frac{3+2}{2} \cdot 10 = 115$

The squirrel is farthest from building A at time $t = 9$; its greatest distance from the building is 140.

2 : { 1 : identifies candidates; 1 : answers }

(c) The total distance traveled is $\int_0^{18} |v(t)|\,dt = 140 + 50 + 25 = 215$.

1 : answer

(d) For $7 < t < 10$, $a(t) = \frac{20 - (-10)}{7 - 10} = -10$

$v(t) = 20 - 10(t - 7) = -10t + 90$

$x(7) = \frac{7+5}{2} \cdot 20 = 120$

$x(t) = x(7) + \int_7^t (-10u + 90)\,du$
$= 120 + \left(-5u^2 + 90u\right)\Big|_{u=7}^{u=t}$
$= -5t^2 + 90t - 265$

4 : { 1 : $a(t)$; 1 : $v(t)$; 2 : $x(t)$ }