

April 25

What is an inverse function?

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Students will verbally explain how to find the derivative of inverse functions

(using the words: coomposite, chain, etc...)

Inverse functions

If $f(x)$ and $g(x)$ are inverse functions:

$$f(g(x)) = x \quad g(f(x)) = x$$

Find $g'(x)$ - the derivative of $f^{-1}(x)$

$$\frac{d}{dx}(f(g(x))) = \frac{f'(g(x))g'(x)}{f'(g(x))} = 1$$

domain of $f(x)$ = range of $g(x)$

$$g'(x) = \frac{1}{f'(g(x))}$$

1. Derivative of an Inverse of a Function

What you are finding: There is probably no topic that confuses students (and teachers) more than inverses. The inverse of a function f is another function f^{-1} that "undoes" what f does. So $f^{-1}(f(x)) = x$. For instance, the inverse of adding 5 is subtracting 5. Start with any number x , add 5, then subtract 5, and you are back to x . Do not confuse the inverse f^{-1} with the reciprocal. $x^{-1} = \frac{1}{x}$ but $(f(x))^{-1} = \frac{1}{f(x)}$.

To find the inverse of a function, you replace x with y and y with x . The inverse to the function $y = 4x - 1$ is $x = 4y - 1$ or $y = \frac{x+1}{4}$.

In this section, you are concerned with finding the derivative of the inverse to a function: $\frac{d}{dx}[f^{-1}(x)]$.

How to find it: The formula used is: $\frac{dy}{dx} = \frac{1}{f'(y)}$. But what I suggest, rather than memorizing this formula, is to switch x and y to find the inverse, and then take the derivative, using implicit differentiation:

$$x = f(y) \Rightarrow 1 = f'(y) \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{f'(y)}$$

Example 72: Find the derivative of the inverse to $y = x^3$ at $x = 8$.

$$f(x) = x^3 \quad g'(x) = \frac{1}{f'(g(x))} \rightarrow g'(8) = \frac{1}{f'(g(8))}$$

$$g'(x) = \frac{1}{3x^2} \quad f'(x) = 3x^2 \quad g(8) \Rightarrow f(x) = 8 \quad x^3 = 8 \quad x = 2$$

Example 74: (Calc) Find the derivative of the inverse to $f(x) = x + 3 \sin x$ at $x = 6$.

Example 75: If $f(x) = x^3$ (1st quadrant), write the equation of the tangent line to $f^{-1}(x)$ at $x = 16$.

Example 76: If $f(x) = x^3 + x^2 + x + 1$, write the equation of the tangent line to $f^{-1}(x)$ at $x = 4$.

Example 77: In the chart to the right, selected values of x are given along with values of $f(x)$ and $f'(x)$.

a) If f^{-1} is the inverse function of f , find the derivative of f^{-1} at $x = -1$: $(f^{-1})'(-1)$.

$$g(x) = f^{-1}(x) \quad g'(-1) = \frac{1}{f'(g(-1))} \quad g(-1) \rightarrow -1 = f(x) \quad x = 0 \quad g(-1) = 0$$

b) Write an equation for the line tangent to the graph of $y = f^{-1}(x)$ at $x = 2$.

$$x = 2 \quad f(2) \rightarrow 2 = f(x) \quad x = 1 \quad f^{-1}(2) = 1 \quad y\text{-value}$$