AB Calculus Fall Review

Get Started!

Jeopardy

Derivatives	Applications of Derivatives	Integrals	Applications of Integrals	Extra Topics
~~100~~	100		100	100
200	200	200		200
~~300~~	300	300		
~~400~~	~~400~~			400
		~~500~~	~~500~~	~~500~~

Team 1 100

Team 2 1300

Take me to Final Jeopardy

Derivatives

If $f(x) = e^{\sin x}$, how many zeros does $f'(x)$ have on the closed interval $[0, 2\pi]$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Derivatives

(B) 2

$$f(x) = e^{\sin x}$$
$$f'(x) = (\cos x)e^{\sin x}$$
$$0 = (\cos x)e^{\sin x}$$
$$0 = \cos x \qquad 0 = e^{\sin x}$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \text{never}$$

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Derivatives

If $f(x) = \sqrt{4\sin x + 2}$, then $f'(0) =$

(A) -2 (B) 0 (C) 1 (D) $\frac{\sqrt{2}}{2}$ (E) $\sqrt{2}$

2000 1900

Derivatives

(E) $\sqrt{2}$

$$f(x) = \sqrt{4\sin x + 2}$$

$$f(x) = (4\sin x + 2)^{\frac{1}{2}}$$

$$f'(x) = \tfrac{1}{2}(4\sin x + 2)^{-\frac{1}{2}}(4\cos x)$$

$$f'(x) = \frac{4\cos x}{2\sqrt{4\sin x + 2}}$$

$$f'(0) = \frac{4\cos 0}{2\sqrt{4\sin 0 + 2}} = \frac{4(1)}{2\sqrt{4(0) + 2}}$$

$$= \frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

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Derivatives

If $y = x(\ln x)^2$, then $\frac{dy}{dx} =$

(A) $3(\ln x)^2$

(B) $(\ln x)(2x + \ln x)$

$\boxed{\text{(C) } (\ln x)(2 + \ln x)}$

(D) $(\ln x)(2 + x\ln x)$

$\boxed{\text{(E) } (\ln x)(1 + \ln x)}$

Derivatives

(C) $(\ln x)(2 + \ln x)$

$$y = x(\ln x)^2$$

$$\frac{dy}{dx} = (1)(\ln x)^2 + 2(\ln x)^1\left(\tfrac{1}{x}\right)(x) =$$

$$(\ln x)^2 + 2\ln x = (\ln x)(\ln x + 2)$$

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Applications of Derivatives

If the graph of $f(x) = 2x^2 + \frac{k}{x}$ has a point of inflection at $x = -1$, then the value of k is

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Applications of Derivatives

(E) 2

$$f(x) = 2x^2 + \frac{k}{x} = 2x^2 + kx^{-1}$$

$$f'(x) = 4x - kx^{-2}$$

$$f''(x) = 4 + 2kx^{-3} = 4 + \frac{2k}{x^3}$$

inflection point at $x = -1$

$$0 = 4 + \frac{2k}{(-1)^3} = 4 - 2k$$

$$4 = 2k \Rightarrow k = 2$$

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Integrals

Let $f(x)$ be the function defined by $f(x) = \begin{cases} x, & x \leq 0 \\ x+1, & x > 0 \end{cases}$

The value of $\int_{-2}^{1} x f(x)\, dx =$

(A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) $\frac{11}{2}$

Integrals

(D) $\frac{7}{2}$

$$f(x) = \begin{cases} x, & x \leq 0 \\ x+1, & x > 0 \end{cases}$$

$$\int_{-2}^{1} x\ f(x)\, dx = \int_{-2}^{0} x\ f(x)\, dx + \int_{0}^{1} x\ f(x)\, dx =$$
$$\int_{-2}^{0} x(x)\, dx + \int_{0}^{1} x(x+1)\, dx =$$
$$\int_{-2}^{0} x^2\, dx + \int_{0}^{1} x^2 + x\, dx =$$

$$\left(\frac{x^3}{3}\Bigg|_{-2}^{0}\right) + \left(\frac{x^3}{3} + \frac{x^2}{2}\Bigg|_{0}^{1}\right) = \left(\frac{0^3}{3} - \frac{(-2)^3}{3}\right) + \left(\left(\frac{1^3}{3} + \frac{1^2}{2}\right) - \left(\frac{0^3}{3} + \frac{0^2}{2}\right)\right) =$$
$$\left(0 + \frac{8}{3}\right) + \left(\left(\frac{1}{3} + \frac{1}{2}\right) - (0+0)\right) = \frac{9}{3} + \frac{1}{2} = 3 + \frac{1}{2} = \frac{7}{2}$$

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Applications of Integrals

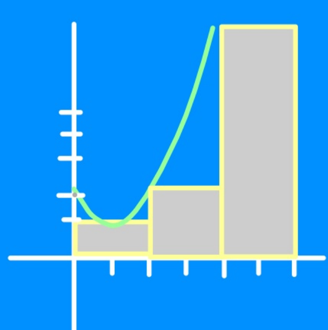
If $\int_0^6 (x^2 - 2x + 2)\, dx$ is approximated by three inscribed rectangles of equal width on the x-axis, then the approximation is

(A) 24 (B) 26 (C) 28 (D) 48 (E) 76

Applications of Integrals

(B) 26

Inscribe Rectangles



Interval	x-value	y-value	Width	Area
0 – 2	1	1	2	2
2 – 4	2	2	2	4
4 – 6	4	10	2	20
			Total Area	26

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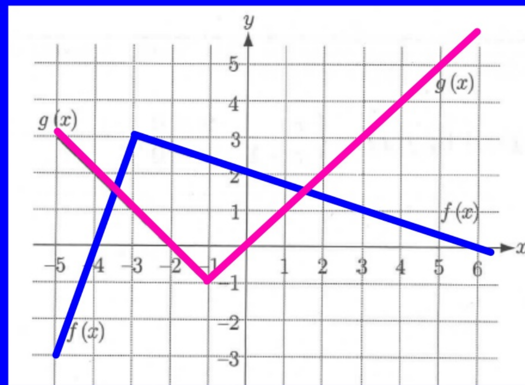
Extra Topics

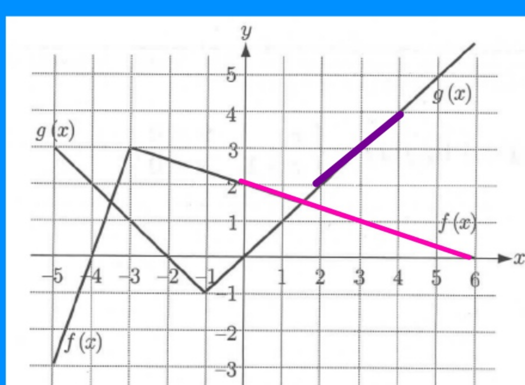


The functions f and g are piecewise linear functions whose graphs are shown above. If $h(x) = f(x)g(x)$, then $h'(3) =$

(A) $-\frac{8}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) $\frac{8}{3}$

Extra Topics

(C) 0



$$h(x) = f(x)g(x)$$
$$h'(x) = f'(x)g(x) + g'(x)f(x)$$
$$h'(3) = f'(3)g(3) + g'(3)f(3)$$

$f(3) = 1$ $g(3) = 3$

$f'(3) =$ slope of f at $x = 3$

$f'(3) = -1/3$

$g'(3) =$ slope of g at $x = 3$

$g'(3) = 1$

$$h'(3) = (-1/3)(3) + 1(1)$$
$$= -1 + 1 = 0$$

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