December 3

If we know the area between the function, g(x), and the x-axis from x = -1 to x = 7 and the area from x = 3 to x = 7,

Describe in words and with symbols how to find the area from x = -1 to x = 3.

$$\int_{-1}^{7} g(x)\,dx = \int_{-1}^{3} g(x)\,dx + \int_{3}^{7} g(x)\,dx$$

$$-\int_{3}^{7} g(x)\,dx \qquad -\int_{3}^{7} g(x)\,dx$$

$$\int_{-1}^{7} g(x)\,dx - \int_{3}^{7} g(x)\,dx = \int_{-1}^{3} g(x)\,dx$$

December 3

Students will verbally explain how to find the exact area under a curve using definite integrals

(using the words: right, left, above, below, antiderivative...)

2. Without changing the value of *a*, how could you use the values of the accumulation function in question 1 to find $\int_0^3 f(t)\,dt$? Explain your thinking.

$$-.6-.6=\int_{-3}^{3} f(t)\,dt-\int_{-3}^{0} f(t)\,dt$$

3. Without changing the value of *a*, use the accumulation function and your thinking from question 2 to find the following. For each, be sure to explain your thinking.

a. $\int_1^4 f(t)dt =$ $\int_{-3}^{4} f(t)dt - \int_{-3}^{1} f(t)dt = A(4)-A(1)$

b. $\int_{-2}^2 f(t)dt =$ $\int_{-3}^{2} f(t)dt-\int_{-3}^{-2} f(t)dt = A(2)-A(-2)$

c. $\int_0^{-1} f(t)dt =$ $\int_{-3}^{-1} f(t)dt-\int_{-3}^{0} f(t)dt = A(-1)-A(0)$

$$A(x)=\int_{-3}^{x} f(x)dx$$

4. The top graph shows the original function, *f*, and the measurement of an accumulation function as the point *x* is dragged along the *t*-axis. The bottom graph shows the accumulation function as a function of *x*. What relationship, if any, do you notice between the original function and the accumulation function? Explain.

5. The top graph on page 1.4 is the graph of the accumulation function, $y = A(x)$, for the function f from the previous pages, and the bottom graph shows the graph of its derivative, $y = A'(x)$.

   a. Choose several values of x and find the corresponding values of $A'(x)$. For each of these, how do they compare to the value of $f(x)$ for that x? What do you observe? Does this make sense? Explain.

   b. Given your response to a, complete the following:
   $f(x)$ is the derivative of $A(x)$.
   $A(x)$ is the original function of $f(x)$.
   antiderivative

$4x + 8$

6. Drag point a on the top graph on page 1.4.

   a. What are you changing in the accumulation function when you change a? What are you changing in the graph of the accumulation function? Explain.
   vertical shift → changing y-values

   b. Using what you know about the accumulation function, why do you think the bottom graph doesn't change when you change the value of a? Explain.

$A(x)$ $f(x)$

derivative $= A'(x)$

$A(x) + K$

↳ derivative of a constant $= 0$

$$f(x) = 4x + 2 = A'(x)$$

$$A(x) = 2x^2 + 2x + K$$

$$\int_1^2 f(x)\,dx = A(2) - A(1)$$

$$\left(2(2)^2 + 2(2) + K\right) - \left(2(1)^2 + 2(1) + K\right)$$

$$= 8$$