December 4

How can you find the exact area under the curve of a function f(x)?

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Students will verbally explain how to find the exact area under a curve using definite integrals

(using the words:
right, left, above, below, antiderivative...)

10. Use your response to question 8 to find $\int_0^3 2x\,dx$. Explain your solution. How can you check your work?

$$\int_0^3 2x\,dx = x^2\Big|_0^3 = 3^2 - 0^2 = 9$$

$$\frac{d}{dx}(?) = 2x$$

$$? = x^2$$

Fundamental Theorem of Calculus

(one part)

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

where $F'(x) = f(x)$

$f(x)$ is the derivative

$F(x)$ is the original function (antiderivative)

$\int_1^3 3x^2\,dx$

$\frac{d}{dx}(?) = 3x^2$

$? = x^3$

$\int_1^3 3x^2\,dx = x^3\Big|_1^3 = 3^3 - 1^3 = 27 - 1 = 26$

$\int_{-1}^2 12x^3\,dx$

$\frac{d}{dx}(?) = 12x^3$

$? = 3x^4$

$\int_{-1}^2 12x^3\,dx = 3x^4\Big|_{-1}^2 = 3(2)^4 - 3(-1)^4$

$= 3(16) - 3(1) = 48 - 3 = 45$

In General

$\int_a^b x^n\,dx$

$\frac{d}{dx}(?) = x^n$

$? = \frac{x^{n+1}}{n+1}$

$\int_a^b x^n\,dx = \frac{x^{n+1}}{n+1}\Big|_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$

$\int_1^4 5x^2 - x\,dx$

$= \int_1^4 5x^2\,dx - \int_1^4 x\,dx$

$= 5\int_1^4 x^2\,dx - \int_1^4 x\,dx = 5\left(\frac{x^3}{3}\Big|_1^4\right) - \left(\frac{x^2}{2}\Big|_1^4\right)$

$\frac{d}{dx}(?) = x^2$

$? = \frac{x^3}{3}$

$\frac{d}{dx}(?) = x^1$

$? = \frac{x^2}{2}$

$= 5\left(\frac{4^3}{3} - \frac{1^3}{3}\right) - \left(\frac{4^2}{2} - \frac{1^2}{2}\right)$

$= 5\left(\frac{64}{3} - \frac{1}{3}\right) - \left(\frac{16}{2} - \frac{1}{2}\right)$

$= 5\left(\frac{63}{3}\right) - \frac{15}{2}$

$= 5(21) - \frac{15}{2} = 97.5$

$\int_0^5 7x^3+2\,dx$

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$= 7\int_0^5 x^3\,dx + \int_0^5 2\,dx$

$\frac{d}{dx}(?)=x^3$ $\qquad$ $\frac{d}{dx}(?)=2$

$? = \frac{x^4}{4}$ $\qquad$ $? = 2x$

$7\int_0^5 x^3\,dx + \int_0^5 2\,dx$

$= 7\left(\frac{x^4}{4}\Big|_0^5\right) + \left(2x\Big|_0^5\right)$

$= 7\left(\frac{5^4}{4} - \frac{0^4}{4}\right) + \left(2(5) - 2(0)\right)$

$7\left(\frac{625}{4}\right) + 10 = 1103.75$

$\int_0^5 7x^3+2\,dx = \frac{7x^4}{4} + 2x\Big|_0^5$

$= \frac{7(5)^4}{4} + 2(5)$