February 10

(Sit in alphabetical order by first name)

How do you evaluate an indefinite integral?

February 10

Students will verbally explain how to find the solution to a differential equation.

(using the words:
separate, initial value, constant ...)

$\frac{dy}{dx} = \frac{1}{x^2} + 5x^4$

find the general solution for the differential equation above ($y =$ ____)

① separate variables

$$\frac{dy}{dx} = \frac{1}{x^2} + 5x^4$$

$$dx\left(\frac{dy}{dx}\right) = \left(\frac{1}{x^2} + 5x^4\right) dx$$

multiply by dx

$$dy = \left(\frac{1}{x^2} + 5x^4\right) dx$$

② integrate both sides (anti-differentiate)

$$\int 1\, dy = \int \frac{1}{x^2} + 5x^4\, dx$$

$$y = \int x^{-2} + 5x^4\, dx$$

$$y = \frac{x^{-1}}{-1} + \frac{5x^5}{5} + C$$

$$y = -\frac{1}{x} + x^5 + C$$

⑤ solve for y

Solve the separable differential equation

$\frac{dy}{dx} = 4x^3 y$ if $y = -2$ when $x = 0$

① separate variables

$$\frac{dy}{dx} = 4x^3 y$$

$$dx\left(\frac{dy}{dx}\right) = (4x^3 y)\, dx$$

$$\frac{dy}{y} = \frac{4x^3 y\, dx}{y}$$

$$\frac{dy}{y} = 4x^3 dx$$

② anti-differentiate

$$\int \frac{dy}{y} = \int 4x^3 dx$$

$$\int \frac{1}{y} dy = \int 4x^3 dx$$

$$\ln|y| = \frac{4x^4}{4} + C$$

$$\ln|y| = x^4 + C$$

③ plug-in initial conditions + solve for C

$x = 0,\ y = -2$

$$\ln|-2| = 0^4 + C$$

$$\ln|-2| = C$$

④ substitute C into equation

$$\ln|y| = x^4 + \ln|-2|$$

⑤ solve for y

$$\ln|y| = x^4 + \ln|-2|$$

$$e^{\ln|y|} = e^{x^4 + \ln|-2|}$$

$$|y| = e^{x^4} \cdot e^{\ln|-2|}$$

$$|y| = |-2|\, e^{x^4}$$

$\hookrightarrow y = -2e^{x^4}$ or $y = 2e^{x^4}$

⑥ check using initial cond.

$-2 \neq 2e^{0^4}$ $\quad -2 = -2e^{0^4}$ True $\quad y = -2e^{x^4}$

Solve the separable differential equation

$\frac{dy}{dx} = \frac{x^2+3}{2y}$

if $y=4$ when $x=1$

① separate

$\frac{dy}{dx} = \frac{x^2+3}{2y}$

$dx\left(\frac{dy}{dx}\right) = \left(\frac{x^2+3}{2y}\right)dx$

$dy = \frac{x^2+3}{2y}\,dx$

$2y\,(dy) = \left(\frac{x^2+3}{2y}\,dx\right)2y$

$2y\,dy = x^2+3\,dx$

② A-D

$\int 2y\,dy = \int x^2+3\,dx$

$y^2 = \frac{x^3}{3}+3x+C$

$y^2 = \frac{x^3}{3}+3x+C$

③ IC

$4^2 = \frac{1^3}{3}+3(1)+C$

$16 = \frac{1}{3}+3+C$

$\frac{48}{3} = \frac{1}{3}+\frac{9}{3}+C$

$\frac{48}{3}-\frac{1}{3}-\frac{9}{3} = C$

$\frac{38}{3} = C$

④ sub in C

$y^2 = \frac{x^3}{3}+3x+\frac{38}{3}$

⑤ solve for y

$y = \pm\sqrt{\frac{x^3}{3}+3x+\frac{38}{3}}$

⑥ check IC

$4 = \sqrt{\frac{1^3}{3}+3(1)+\frac{38}{3}}$

$y = \sqrt{\frac{x^3}{3}+3x+\frac{38}{3}}$

$\frac{dy}{dx} = \frac{\cos(x)}{3y^2}$

$y=1,\ x=\frac{\pi}{2}$