February 14

What is your favorite part of Valentine's Day?

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Students will verbally explain how to find the solution to a differential equation

(using the words: separate, initial value, constant ...)

Exponential Growth

Consider the statement "The rate of change of some quantity y is directly proportional to y"

This is like saying that the more money you have (y), the faster it will grow $\left(\frac{dy}{dt}\right)$, or the more you scratch an insect bite, the worse it will get, or the more addicted someone is to a substance, the more the addiction will grow.

"The rate of change of some quantity y is directly proportional to y" can be translated:

Quantity $= y$ **Rate of change of** $y = \frac{dy}{dt}$ **Directly proportional** = multiplied by some constant k

So this statement can be translated: $\frac{dy}{dt} = ky$ which can be translated into $y =$ Ae^{kt}.

Example A: *Punctured Tire Problem:* You run over a nail. As the air leaks out of your tire, the rate of change of air pressure inside the tire is directly proportional to that pressure.

(a) Write a differential equation that states that fact if the pressure is 35 lbs/psi and decreasing at the rate of 0.28 lbs/psi/min at the time the nail is struck.

$\frac{dP}{dt} = kP$ $\quad -.28 = k35$ $\quad k = \frac{-.28}{35} = -.008$ $\quad \frac{dP}{dt} = -.008P$

(b) Solve the differential equation

$\frac{dP}{dt} = -.008P$ $\quad P = Ae^{-.008t} = 35e^{-.008t}$

$t=0, P=35$

(c) What will the pressure be at 10 minutes after the tire was punctured?

$P = 35e^{-.008(10)} = 32.3$ lbs/psi

(d) The care is safe to drive as long as the pressure is 12 lbs/psi or greater. For how long after the puncture will the car be safe to drive?

$\frac{12}{35} = \frac{35e^{-.008t}}{35} \rightarrow .342 = e^{-.008t} \rightarrow \frac{\ln(.342)}{-.008} = \frac{-.008t}{-.008}$

$\ln(.342) = \ln(e^{-.008t})$

$t = 134.118$ min

Remember that you can always translate $\frac{dy}{dt} = ky$ into $y = Ce^{kt}$ if and only if the original statement is:

The rate of change of some quantity y is directly proportional to y. Suppose it isn't? For example:

a) the rate of change of y is proportional to $4y$.

$\frac{dy}{dt} = K(4y)$

b) the rate of change of y is proportional to $4 - y$.

$\frac{dy}{dt} = K(4-y)$

$\frac{dy}{4-y} = \frac{K(4-y)dt}{4-y} \rightarrow \frac{dy}{4-y} = K\,dt$

$\frac{dy}{dt} = y' = 4y + 24 = 4(y+6)$

c) the rate of change of y is inversely proportional to y.

$\frac{dy}{dt} = K\left(\frac{1}{y}\right)$

d) the rate of change of y is proportional to $\sqrt{y}$

$\frac{dy}{dt} = K\sqrt{y}$