February 24

How do you find area between two curves?

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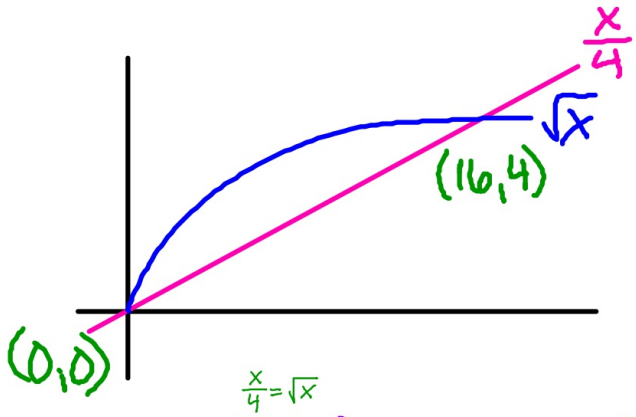
Students will verbally explain how to find the area bounded by two functions

(using the words:
above, below, right, left...)

The area of the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{4}$



$\frac{x}{4} = \sqrt{x}$

$(x)^2 = (4\sqrt{x})^2$

$x^2 = 16x$

$x^2 - 16x = 0$

$x(x-16) = 0$

$x = 0 \quad x - 16 = 0$

$x = 16$

$$A = \int_0^{16} \sqrt{x} - \frac{x}{4}\,dx$$

$\frac{1}{4}x$

$$= \int_0^{16} x^{\frac{1}{2}} - \frac{x}{4}\,dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{4}\cdot\frac{x^2}{2}\Big|_0^{16}$$

$$= \frac{2}{3}\sqrt{x}^3 - \frac{x^2}{8}\Big|_0^{16}$$

$$= \frac{2}{3}\left(\sqrt{16}\right)^3 - \frac{16^2}{8} - \left(\frac{2}{3}\sqrt{0}^3 - \frac{0^2}{8}\right)$$

$$= \frac{2}{3}(64) - 32 = \frac{128}{3} - \frac{96}{3} = \frac{32}{3}$$

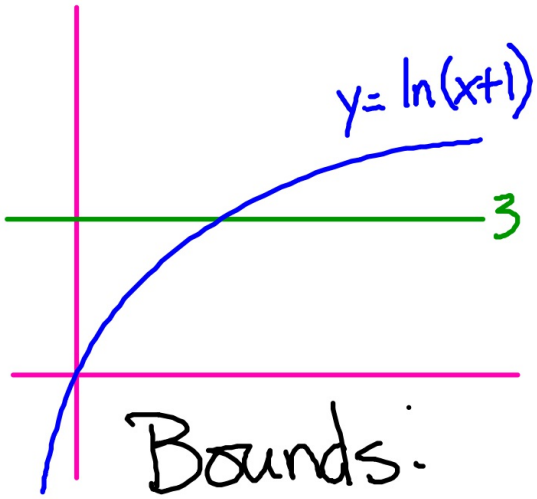
Find the area of the region in the first quadrant enclosed by $y = \ln(x+1)$, $y = 3$, and the y-axis.

$y = \ln(x+1)$

$e^y = e^{\ln(x+1)}$

$e^y = x + 1$

$e^y - 1 = x$



Bounds:

$y = 0, \; y = 3$

$$A = \int_0^3 e^y - 1\,dy$$

$$= e^y - y\Big|_0^3$$

$$= e^3 - 3 - (e^0 - 0)$$

$$= e^3 - 3 - 1 = e^3 - 4$$

Find the area of the region in the first quadrant enclosed by the graphs of $y = 3x$ and $y = x^2$.

$x^2 = 3x$

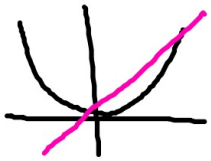
$x = \sqrt{3x}$

$x^2 - 3x = 0$

$(x)(x-3) = 0$

$x = 0$

$x = 3$



$$\int_0^3 3x - x^2 \, dx = \frac{3x^2}{2} - \frac{x^3}{3}\Big|_0^3$$

$$\left(\frac{3(3)^2}{2} - \frac{(3)^3}{3}\right) - \left(\frac{3(0)^2}{2} - \frac{0^3}{3}\right)$$

$13.5 - 9 = 4.5$