

February 4

What is the difference between a definite integral and an indefinite integral?

February 4

Students will verbally explain how to evaluate definite integrals by substitution

(using the words:
inside, outside, product, differential, terms...)

$$\int_0^2 (4x-7)^5 dx$$

$$u = 4x - 7$$

$$du = 4 dx$$

$$\frac{du}{4} = dx$$

$$\int_0^2 u^5 \left(\frac{du}{4}\right)$$

in terms of u

x -values

(we need to change to u)

$$\text{if } x=0, u=4(0)-7=-7$$

$$\text{if } x=2, u=4(2)-7=1$$

$$\begin{aligned} &\rightarrow \int_{u=-7}^{u=1} u^5 \left(\frac{du}{4}\right) \\ &= \frac{1}{4} \int_{-7}^1 u^5 du \\ &= \frac{1}{4} \left(\frac{u^6}{6} \Big|_{-7}^1 \right) \\ &= \frac{1}{4} \left(\frac{(1)^6}{6} - \frac{(-7)^6}{6} \right) \\ &= \frac{1}{24} - \frac{7^6}{24} \end{aligned}$$

$$\int_4^6 \frac{4x}{x^2-10} dx$$

$$u = x^2 - 10$$

$$du = 2x dx$$

$$\frac{du}{2x} = dx$$

$$x=4 \rightarrow u=4^2-10=6$$

$$x=6 \rightarrow u=6^2-10=26$$

$$\begin{aligned} &\int_6^{26} \frac{4x}{u} \frac{du}{2x} \\ &= \int_6^{26} \frac{2}{u} du \\ &= 2 \int_6^{26} \frac{1}{u} du \end{aligned}$$

$$= 2 \left(\ln u \Big|_6^{26} \right)$$

$$= 2 (\ln 26 - \ln 6)$$

$$= 2 \ln \left(\frac{26}{6} \right) = \ln \left(\frac{26}{6} \right)^2$$

$$\int_0^3 x\sqrt{25-x^2} dx$$

$$u = 25 - x^2$$

$$du = -2x dx$$

$$\frac{du}{-2x} = dx$$

$$x=3 \rightarrow u = 25 - 3^2 = 16$$

$$x=0 \rightarrow u = 25 - 0^2 = 25$$

Pg 335

#77, 78

79-89 (odd)

$$\begin{aligned} \int_{25}^{16} x\sqrt{u} \left(\frac{du}{-2x} \right) &= -\frac{1}{2} \int_{25}^{16} \sqrt{u} du && \leftarrow \text{flip bounds} \\ &= -\left(-\frac{1}{2} \int_{16}^{25} \sqrt{u} du \right) && \text{+ multiply by } -1 \\ &= \frac{1}{2} \int_{16}^{25} \sqrt{u} du \\ &= \frac{1}{2} \int_{16}^{25} u^{1/2} du \\ &= \frac{1}{2} \left(\frac{u^{3/2}}{3/2} \Big|_{16}^{25} \right) = \frac{1}{2} \left(\frac{2u^{3/2}}{3} \Big|_{16}^{25} \right) = \frac{1}{2} \left(\frac{2\sqrt{u}^3}{3} \right) \Big|_{16}^{25} \\ &= \frac{1}{2} \left(\frac{2\sqrt{25}^3}{3} - \frac{2\sqrt{16}^3}{3} \right) = \frac{1}{2} \left(\frac{250}{3} - \frac{128}{3} \right) \\ &= \frac{1}{2} \left(\frac{122}{3} \right) = \frac{61}{3} \end{aligned}$$