

JANUARY 14

How are Riemann Sums  
(LRAM, RRAM, MRAM)  
related to finding the area  
between two curves?

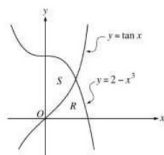
JANUARY 13

Students will verbally explain how to  
find the area bounded by two  
functions

(using the words:  
above, below, right, left...)

1.

2001-01



Let  $R$  and  $S$  be the regions in the first quadrant shown in the figure above. The region  $R$  is bounded by the  $x$ -axis and the graphs of  $y = 2 - x^3$  and  $y = \tan x$ . The region  $S$  is bounded by the  $y$ -axis and the graphs of  $y = 2 - x^3$  and  $y = \tan x$ .

(a) Find the area of  $S$ .

(b) Find the area of  $R$ .

2.

1998-01

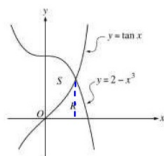
Let  $R$  be the region bounded by the  $x$ -axis, the graph of  $y = \sqrt{x}$ , and the line  $x = 4$ .

(a) Find the area of the region  $R$ .

(b) Find the value of  $h$  such that the vertical line  $x = h$  divides the region  $R$  into two regions of equal area.

1.

2001-01



Let  $R$  and  $S$  be the regions in the first quadrant shown in the figure above. The region  $R$  is bounded by the  $x$ -axis and the graphs of  $y = 2 - x^3$  and  $y = \tan x$ . The region  $S$  is bounded by the  $y$ -axis and the graphs of  $y = 2 - x^3$  and  $y = \tan x$ .

(a) Find the area of  $S$ .

$$2 - x^3 = \tan x$$

$$x = 0.90215505$$

$$\int_0^{0.90215505} \tan x (2 - x^3) dx = 1.1605$$

$$\int_0^{0.902} f(x) - g(x) dx$$

(b) Find the area of  $R$ .

$$\int_0^{0.90215505} \tan x dx + \int_{0.90215505}^{1.259021} 2 - x^3 dx = 0.729$$

$$y = \tan x \rightarrow x = \tan^{-1} y$$

$$y = 2 - x^3 \rightarrow x = \sqrt[3]{2 - y}$$

$$\int_0^{1.259} \sqrt[3]{2 - y} - \tan^{-1} y dy = 0.7293$$

3 points:

- 1 – bounds
- 1 – integrand
- 1 – answer

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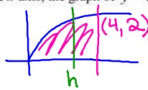
2.

Let  $R$  be the region bounded by the  $x$ -axis, the graph of  $y = \sqrt{x}$ , and the line  $x = 4$ .

1998-21

(a) Find the area of the region  $R$ .

$$\int_0^4 \sqrt{x} \, dx = 5.333$$



2 points:  
1 – integral  
1 – answer

(b) Find the value of  $h$  such that the vertical line  $x = h$  divides the region  $R$  into two regions of equal area.

$$\int_h^4 \sqrt{x} \, dx = \frac{5.333}{2}$$

$$\int_0^h x^{\frac{1}{2}} \, dx = 2.667$$

$$\left. \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right|_0^h = \frac{2}{3} (\sqrt{x})^3 \Big|_0^h = 2.667$$

$$\frac{2}{3} (\sqrt{h})^3 - \frac{2}{3} (\sqrt{0})^3 = 2.667$$

$$\frac{2}{3} (\sqrt{h})^3 = 2.667$$

$$(\sqrt{h})^3 = 4$$

$$\sqrt{h} = 4^{\frac{1}{3}} \rightarrow h = \left( 4^{\frac{1}{3}} \right)^2 = 4^{\frac{2}{3}} = 2.5198$$

2 points:  
1 – equation in  $h$   
1 – answer

$$1 + \frac{1}{2} = \frac{3}{2}$$

$$x^{\frac{3}{2}} = \left( x^{\frac{1}{2}} \right)^3$$