January 21

Explain how to calculate an integral by hand.

$$\int_4^{10} x^2 - 3x \, dx$$

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Students will verbally explain how to evaluate indefinite integrals

(using the words: antiderivative, constant...)

Name: ______________________________

Indefinite Integrals

A. Find the derivative of each of the functions below:

1. $y = x^4 + x^2 + 10$ ______________________

2. $y = -15 + x^2 + x^4$ ______________________

3. $y = x^2 - 23 + x^4$ ______________________

4. $y = x^4 + \pi + x^2$ ______________________

5. What do you notice about all your derivatives?

   $y' = 4x^3 + 2x$ all the same

6. Why did you get the same derivative for all of the functions above?

B. Give three possible ANTI-DERIVATIVES for the function $y = 5x^4$.

$x^5 = \frac{5x^5}{5}$, $x^5 + 5$

$x^5 + 1000$

$x^5 - 1{,}000{,}000$

Indefinite Integrals

$$\int f(x)\,dx = F(x) + C$$

no bounds

where $F'(x) = f(x)$

add an arbitrary constant

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln(a)} + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C$$

$$\int \tan x \, dx =$$

$$\int \cot x \, dx =$$

$$\int \sec x \, dx =$$

$$\int \csc x \, dx =$$

$$\int K \cdot f(x) \, dx = K \int f(x) \, dx$$

$$\int -f(x) \, dx = -\int f(x) \, dx$$

$$\int f(x) \pm g(x) \, dx = \int f(x) dx \pm \int g(x) dx$$

C. Evaluate the following indefinite integrals:

1. $\int 7x^6 \, dx =$ $\frac{7x^7}{7} + C = x^7 + C$

2. $\int \cos(x) \, dx =$ $\sin x + C$

3. $\int e^x + x^3 \, dx =$ $e^x + \frac{x^4}{4} + C$

4. $\int \sin(x) + 6x^2 \, dx =$ $-\cos x + 2x^3 + C$

5. $\int 5 + x - e^x \, dx =$ $5x + x^2/2 - e^x + C$

6. $\int 10x^4 + \csc^2(x) \, dx =$ $\frac{10x^5}{5} - \cot x + C = 2x^5 - \cot x + C$

7. $\int \left[\cot(x)\csc(x) + 7x^6 - \frac{1}{x^2} \right] dx =$ $=x^{-2}$

$-\csc x + \frac{7x^7}{7} - \frac{x^{-1}}{-1} + C = -\csc x + x^7 + \frac{1}{x} + C$

8. $\int \left(x^{-3} + \frac{1}{x^2} - \frac{1}{x} + 10 \right) dx =$ $x^{-2}/{-2} + x^{-1}/{-1} - \ln|x| + 10x + C = -\frac{1}{2x^2} - \frac{1}{x} - \ln|x| + 10x + C$

9. $\int \sqrt{x} + e^x \, dx =$ $\frac{x^{1.5}}{1.5} + e^x + C = \frac{2}{3}x^{3/2} + e^x + C$

10. $\int \sqrt[4]{x^3} - \sin(x) + x^4 \, dx =$ $\frac{4x^{7/4}}{7} + \cos(x) + x^5/5 + C$

$\int x^{\frac{3}{4}} - \sin x + x^4 \, dx$

D. Why don't we need "+ c" when working with a definite integral (one with bounds)?