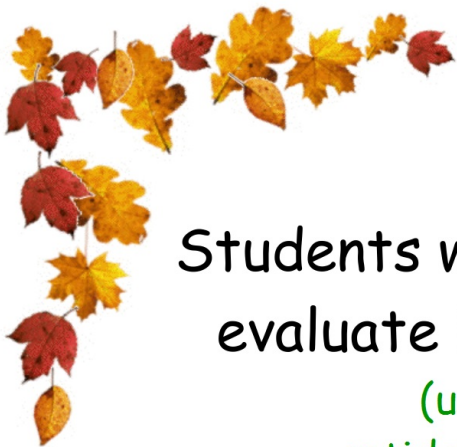


January 21

Explain how to calculate an  
integral by hand.

$$\int_4^{10} x^2 - 3x \, dx$$



January 21

Students will verbally explain how to  
evaluate indefinite integrals

(using the words:  
antiderivative, constant...)



Name: \_\_\_\_\_

### Indefinite Integrals

A. Find the derivative of each of the functions below:

1.  $y = x^4 + x^2 + 10$  \_\_\_\_\_

2.  $y = -15 + x^2 + x^4$  \_\_\_\_\_

3.  $y = x^2 - 23 + x^4$  \_\_\_\_\_

4.  $y = x^4 + \pi + x^2$  \_\_\_\_\_

5. What do you notice about all your derivatives?

$$y' = 4x^3 + 2x \quad \text{all the same}$$

6. Why did you get the same derivative for all of the functions above?

B. Give three possible ANTI-DERIVATIVES for the function  $y = 5x^4$ .

$$x^5 = \frac{5x^5}{5}, \quad \begin{array}{l} x^5 + 5 \\ x^5 + 1000 \\ x^5 - 1,000,000 \end{array}$$

## Indefinite Integrals

$$\int f(x) dx = F(x) + C$$

no bounds

where  $F'(x) = f(x)$

add an  
arbitrary  
constant

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln(a)} + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C$$

$$\int \tan x dx =$$

$$\int \cot x dx =$$

$$\int \sec x dx =$$

$$\int \csc x dx =$$

$$\int k \cdot f(x) dx = k \int f(x) dx$$

$$\int -f(x) dx = - \int f(x) dx$$

$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$$

C. Evaluate the following indefinite integrals:

1.  $\int 7x^6 dx =$

$$\frac{7x^7}{7} + C = x^7 + C$$

2.  $\int \cos(x) dx =$

$$\sin x + C$$

3.  $\int e^x + x^3 dx =$

$$e^x + \frac{x^4}{4} + C$$

4.  $\int \sin(x) + 6x^2 dx =$

$$-\cos x + 2x^3 + C$$

5.  $\int 5 + x - e^x dx =$

$$5x + \frac{x^2}{2} - e^x + C$$

6.  $\int 10x^4 + \csc^2(x) dx =$

$$\frac{10x^5}{5} - \cot x + C = 2x^5 - \cot x + C$$

7.  $\int \left[ \cot(x) \csc(x) + 7x^6 - \frac{1}{x^2} \right] dx =$

$$\overset{=x^{-2}}{\left[ \cot(x) \csc(x) + 7x^6 - \frac{1}{x^2} \right]} dx = -\csc x + \frac{7x^7}{7} - \frac{x^{-1}}{-1} + C = -\csc x + x^7 + \frac{1}{x} + C$$

8.  $\int \left( x^{-3} + \frac{1}{x^2} - \frac{1}{x} + 10 \right) dx =$

$$\frac{x^{-2}}{-2} + \frac{x^{-1}}{-1} - \ln|x| + 10x + C = -\frac{1}{2x^2} - \frac{1}{x} - \ln|x| + 10x + C$$

9.  $\int \sqrt{x} + e^x dx =$

$$\frac{x^{1.5}}{1.5} + e^x + C = \frac{2}{3}x^{3/2} + e^x + C$$

10.  $\int \sqrt[4]{x^3} - \sin(x) + x^4 dx =$

$$\frac{4x^{7/4}}{7} + \cos(x) + \frac{x^5}{5} + C$$

$$\int x^{3/4} - \sin x + x^4 dx$$

D. Why don't we need "+C" when working with a definite integral (one with bounds)?