

JANUARY 7

What is the difference between  
a derivative and an integral?

↓  
rate

↑  
amount

ARE YOU COMING TO  
THE STUDY SESSION ON  
SATURDAY??



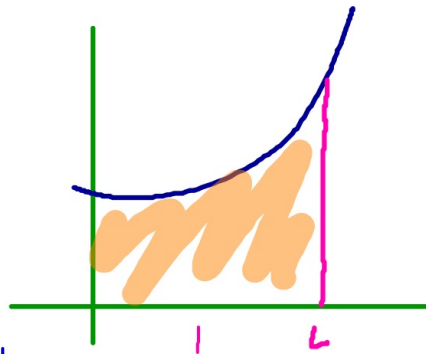
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Students will verbally explain how to  
find the area bounded by two  
functions

(using the words:  
above, below, right, left...)

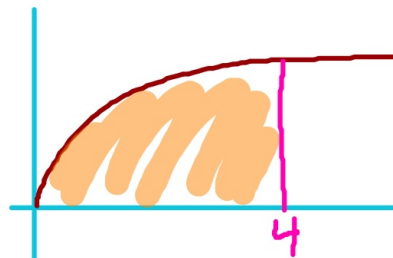


Find the area  
bounded by  
 $f(x) = 0.5x^2 + 3$   
 $x = 4$   
and the y-axis  
+  $y = 0$



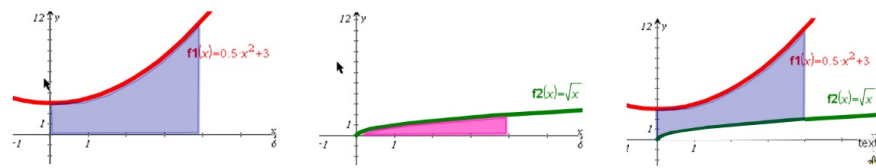
$$\begin{aligned}
 & \int_0^4 0.5x^2 + 3 \, dx \\
 &= 0.5 \int_0^4 x^2 \, dx + \int_0^4 3 \, dx \\
 &= 0.5 \left( \frac{x^3}{3} \right) \Big|_0^4 + 3x \Big|_0^4 \\
 &= 0.5 \left( \frac{4^3}{3} - \frac{0^3}{3} \right) + 3(4 - 0) \\
 &= \frac{1}{2} \left( \frac{64}{3} \right) + 12 = \frac{32}{3} + 12 = \frac{32}{3} + \frac{36}{3} = \frac{68}{3}
 \end{aligned}$$

Find the area  
bounded by  
 $g(x) = \sqrt{x}$   
 $x = 4$   
and the y-axis  
+  $y = 0$



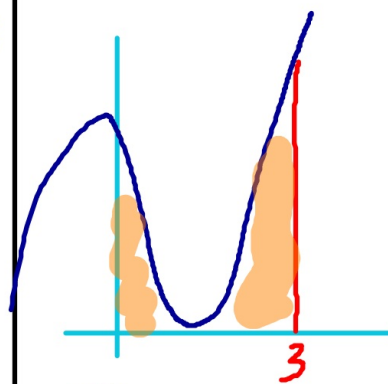
$$\begin{aligned}
 & \int_0^4 \sqrt{x} \, dx = \int_0^4 x^{1/2} \, dx \\
 &= \frac{x^{3/2}}{\frac{3}{2}} \Big|_0^4 = \frac{2}{3} x^{3/2} \Big|_0^4 \\
 &= \frac{2}{3} (4)^{3/2} - \frac{2}{3} (0)^{3/2} \\
 &= \frac{2}{3} (\sqrt{4})^3 = \frac{2}{3} (2)^3 = \frac{2}{3} (8) = \frac{16}{3}
 \end{aligned}$$

Find the area  
bounded by  
 $f(x) = 0.5x^2 + 3$   
 $g(x) = \sqrt{x}$   
 $x = 4$   
and the y-axis



$$\int_0^4 \underbrace{.5x^2 + 3}_{\text{top function}} - \underbrace{\sqrt{x}}_{\text{bottom function}} dx = \frac{52}{3}$$

Find the area  
bounded by  
 $f(x) = 0.5x^3 - 6x + 8$   
the x-axis and the  
lines  $x = 0$  and  $x = 3$



$$\int_0^3 .5x^3 - 6x + 8 dx = 7.125$$

Find the area

bounded by

$$g(x) = -0.25x^2 + x + 11$$

the x-axis and the

lines  $x = 0$  and  $x = 3$

$$\int_0^3 -.25x^2 + x + 11 dx = 35.25$$

Find the area

bounded by

$$g(x) = -0.25x^2 + x + 11$$

$$f(x) = 0.5x^3 - 6x + 8$$

and the lines  $x = 0$

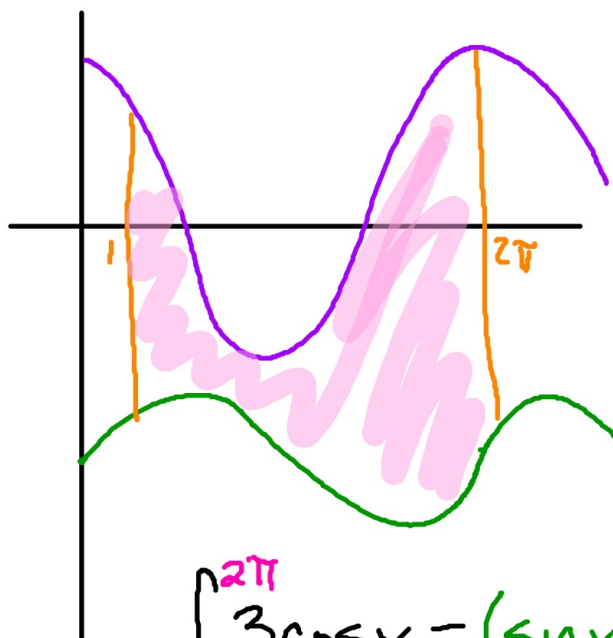
and  $x = 3$

$$28.125 = \int_0^3 g(x) - f(x) dx$$

# Area between two curves

$$\int_a^b \text{upper function} - \text{lower function} \, dx$$

Find the area  
bounded by  
 $f(x) = 3\cos x$   
 $g(x) = \sin x - 4$   
on the interval  
 $(1, 2\pi)$



$$\int_1^{2\pi} 3\cos x - (\sin x - 4) \, dx$$
$$= 19.068$$