

March 10

How is finding the volume of a given object different from finding the area of a given region?

(Include differences in the problem set up as well as differences in the answer.)

$$\int (\text{area}) dx$$

$$\int \text{function } dx$$

March 10

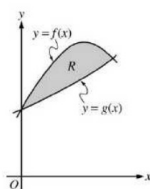
Students will verbally explain how to find the area and volume

(using the words:
cross-section, area, dimensions, slice...)

2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

- Find the area of R .
- Find the volume of the solid generated when R is revolved about the x -axis.
- The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

WRITE ALL WORK IN THE TEST BOOKLET.

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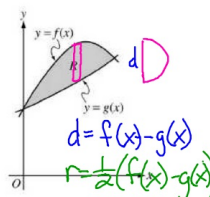
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AP[®] CALCULUS AB
2005 SCORING GUIDELINES (Form B)

Question 1

Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

- Find the area of R .
- Find the volume of the solid generated when R is revolved about the x -axis.
- The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.



The graphs of f and g intersect in the first quadrant at $(S, T) = (1.13569, 1.76446)$.

1 : correct limits in an integral in (a), (b), or (c)

$$\begin{aligned} \text{(a) Area} &= \int_0^S (f(x) - g(x)) \, dx \\ &= \int_0^S (1 + \sin(2x) - e^{x/2}) \, dx \\ &= 0.429 \end{aligned}$$

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^S ((f(x))^2 - (g(x))^2) \, dx \\ &= \pi \int_0^S ((1 + \sin(2x))^2 - (e^{x/2})^2) \, dx \\ &= 4.266 \text{ or } 4.267 \end{aligned}$$

3 : $\begin{cases} 2 : \text{integrand} \\ (-1) : \text{each error} \\ \text{Note: } 0/2 \text{ if integral not of form } \int_a^b (R^2(x) - r^2(x)) \, dx \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(c) Volume} &= \int_0^S \frac{\pi}{2} \left(\frac{f(x) - g(x)}{2} \right)^2 \, dx \\ &= \int_0^S \frac{\pi}{2} \left(\frac{1 + \sin(2x) - e^{x/2}}{2} \right)^2 \, dx \\ &= 0.077 \text{ or } 0.078 \end{aligned}$$

$$A = \frac{\pi^2}{2} \int_0^S \left(\frac{f(x) - g(x)}{2} \right)^2 \, dx$$

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