

March 24

Mock Exam:
March 27th - 7:30 am
Room 5

How do you find the average value
of a function?

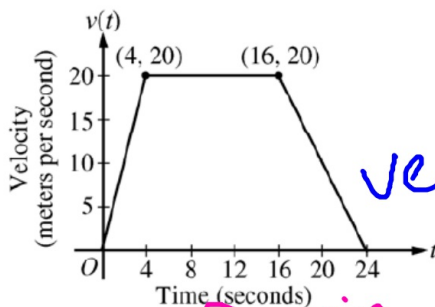
How do you find the average rate of
change?

March 24

Students will verbally explain how to
find the average value and the average
rate of change of a function

(using the words:
integral, interval...)

$$a(t) = \begin{cases} \text{---} \\ \text{---} \\ \text{---} \end{cases}$$



velocity

$$\frac{f(b)-f(a)}{b-a} = f'(c) \quad \text{MVT}$$

5. A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t) dt$. Using correct units, explain the meaning of $\int_0^{24} v(t) dt$. Area meters

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure. Derivative Slope m/s

(c) Let $a(t)$ be the car's acceleration at time t , in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$. Slopes

(d) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c , for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

$$\text{slope} = \frac{f(b)-f(a)}{b-a} \quad a=8 \quad b=20$$

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Question 5

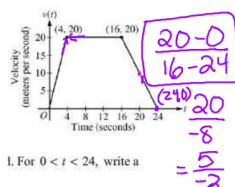
A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t) dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t , in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c , for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?



1. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

Mean Value Theorem guarantee a value of c , for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

(a) $\int_0^{24} v(t) dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$
The car travels 360 meters in these 24 seconds.

(b) $v'(4)$ does not exist because

$$\lim_{t \rightarrow 4^+} \left(\frac{v(t) - v(4)}{t - 4} \right) = 5 \neq 0 = \lim_{t \rightarrow 4^-} \left(\frac{v(t) - v(4)}{t - 4} \right)$$

$$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2} \text{ m/sec}^2$$

(c) $a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$

$a(t)$ does not exist at $t = 4$ and $t = 16$.

$$\frac{0-20}{20-8} = -\frac{10}{16}$$

(d) The average rate of change of v on $[8, 20]$ is

$$\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6} \text{ m/sec}^2$$

No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.

2: { 1: value
1: meaning with units

3: { 1: $v'(4)$ does not exist, with explanation
1: $v'(20)$
1: units

2: { 1: finds the values 5, 0, $-\frac{5}{2}$
1: identifies constants with correct intervals

2: { 1: average rate of change of v on $[8, 20]$
1: answer with explanation