November 19

What do you need to write an equation of a tangent line?
How do you know if using the tangent line will give you an over-estimate or an under-estimate?

November 19

Students will verbally explain how to find the exact area under a curve using definite integrals

(using the words:
right, left, above, below, antiderivative…)

Definite Integral
Student Activity

Name _____
Class _____

Open the TI-Nspire document *Definite_Integral*.

In this activity, you will use a graphical representation to explore the definite integral of a continuous function. You will change the upper and lower limits, *a* and *b*, of the integral $\int_a^b f(x)\,dx$ and observe the resulting changes in the graph and the value of the definite integral.

Move to page 1.2.

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

1. The graph shown is of the function $y = f(x)$. The definite integral of $f(x)$ from a to b is given by $\int_a^b f(x)\,dx$. For example, $\int_0^2 f(x)\,dx$ is the definite integral of $f(x)$ from 0 to 2, or between $x = 0$ and $x = 2$.

   Drag points *a* and *b* along the *x*-axis to determine the values of the following definite integrals, where *f* is the function shown in the graph.

   a. $\int_0^2 f(x)\,dx =$ _____

   b. $\int_{-4}^2 f(x)\,dx =$ _____

   c. $\int_{-4}^{-2} f(x)\,dx =$ _____

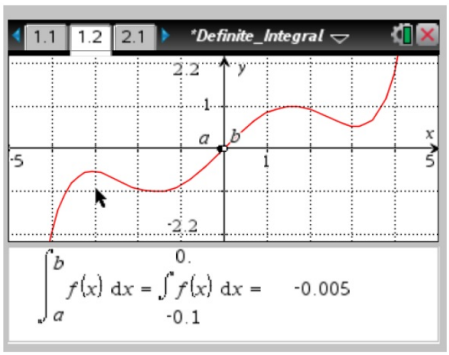
2. Drag point *a* to –3 and move point *b* to determine the following:

   a. For what values of *b* is $\int_{-3}^b f(x)\,dx$ positive? What do you observe about the shaded region and the graph of *f* when $\int_{-3}^b f(x)\,dx$ is positive?

   b. For what values of *b* is $\int_{-3}^b f(x)\,dx$ negative? What do you observe about the shaded region and the graph of *f* when $\int_{-3}^b f(x)\,dx$ is negative?

   c. For what values of *b* does $\int_{-3}^b f(x)\,dx = 0$? What do you observe about the shaded region and the graph of *f* when $\int_{-3}^b f(x)\,dx = 0$?

3. For the function *f* pictured on page 1.2, under what conditions of *a* and *b* in [–5, 5] will the definite integral $\int_a^b f(x)\,dx$ be positive? Negative? Zero? Explain your thinking.

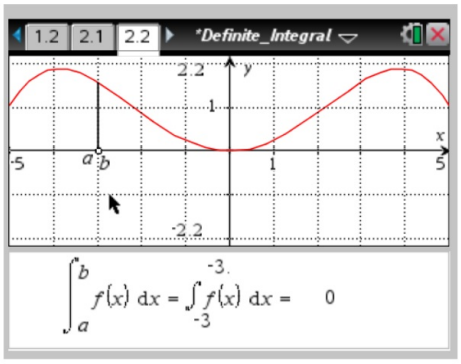


Positive:
- when $b > a$ & $a \geq 0$
- when $b < a$ & $a \leq 0$

Negative
- when $b > a$ & $b \leq 0$
- when $b < a$ & $b \geq 0$

Zero: $|a| = |b|$

5. For the function $f(x)$ pictured on page 2.2, under what conditions of a and b in [–5, 5] will the definite integral $\int_a^b f(x)\,dx$ be positive? Negative? Zero? Explain your thinking.



6. Based on your observations on pages 1.2 and 2.2, for any continuous function f on an interval [c, d] and for a and b in [c, d], when will the definite integral $\int_a^b f(x)\,dx$ be positive? Negative? Zero? Clearly explain your generalization.

Positive:
- $b > a$ and $f(x) > 0$
- $b < a$ and $f(x) < 0$

Negative:
- $b < a$ and $f(x) > 0$
- $b > a$ and $f(x) < 0$

Zero:
- $a = b$
- if area below x-axis = area above x-axis

7. The definite integral $\int_a^b f(x)\,dx$ is often described as "the area under the curve $y = f(x)$ between $x = a$ and $x = b$." What problems do you see with this definition?

Order of Integration	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
Zero	$\int_a^a f(x)\,dx = 0$
Constant Multiple	$\int_a^b k \cdot f(x)\,dx = k\int_a^b f(x)\,dx$ For any number k
Additivity	$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
Constant Multiple (special case)	$\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx \qquad k = -1$
Sum and Difference	$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Quotient Rule

Product Rule