October 1

Describe what a graph will look like if the function has a positive slope. What if the function has a negative slope?

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Students will verbally explain how to use the derivative to give characteristics of a function

(using the words:
increasing, decreasing, positive, negative, zero...)

$$\frac{4x(\cos x + x)}{x^2+1} = g(x)$$

$$\left[4(\cos x + x) + (-\sin x + 1)(4x)\right](x^2+1) - \ldots$$

4. AB Calculus – Step-by-Step Name ______________________________

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	-3	5	2
2	-3	-1	4	6
3	π	8	-1	4
4	-5	Unknown	0	3

The functions *f* and *g* are differentiable for all real numbers *g*. The table above gives values of the function and their first derivatives at selected values of x.

a. If the function h is given by $h(x) = \frac{f(x)}{g(x)} + x$, find $h'(1)$.

$$h(x) = \frac{f(x)}{g(x)} + x$$

$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2} + 1$$

$$h'(1) = \frac{f'(1)g(1) - g'(1)f(1)}{g(1)^2} + 1 = \frac{-3(5)-2(4)}{(5)^2} + 1 = \frac{-15-8}{25} + 1 = \frac{-23}{25} + \frac{25}{25} = \frac{2}{25}$$

b. If the function r is given by $r(x) = (-2f(x))g(x)$, find the equation of the tangent line to $r(x)$ at $x = 2$.

$r(x) = -2f(x)g(x)$

$r'(x) = -2f'(x)g(x) + g'(x)(-2f(x))$

$r'(2) = -2f'(2)g(2) + g'(2)(-2f(2)) = -2(-1)(4) + 6(-2)(-3) = 8 + 36 = 44$ slope

$r(2) = -2f(2)g(2) = -2(-3)(4) = 24$

x-value y-value $y - 24 = 44(x - 2)$

c. If the function v is given by $v(x) = \frac{f(x)-1}{f(x)}$, find the slope of the line parallel to v at $x = 3$.

$$v(x) = \frac{f(x)-1}{f(x)}$$

$$v'(x) = \frac{f'(x)\cdot f(x) - f(x)(f(x)-1)}{f(x)^2}$$

$$v'(3) = \frac{f'(3)\cdot f(3) - f'(3)(f(3)-1)}{f(3)^2} = \frac{8(\pi) - 8(\pi-1)}{\pi^2} = \frac{8\pi - 8\pi + 8}{\pi^2} = \frac{8}{\pi^2}$$

d. If the function w is given by $w(x) = xf(x)$ and $w'(4) = 9$, find $f'(4)$.

$w(x) = xf(x)$

$w'(x) = 1f(x) + f'(x)x$

$w'(4) = 1f(4) + f'(4)4 = 9$

$f(4) + 4f'(4) = 9$

$-5 + 4f'(4) = 9$

$4f'(4) = 14$

$f'(4) = \frac{14}{4}$

$f'(4) = \frac{7}{2}$

Relative Maximum highest y-value/point on an interval	high point any # A function f is said to have a relative maximum at $x = c$ if there is an endpoints are not included: $a < x < b$ or (a, b) open interval containing c on which $f(c)$ is the largest value $a < c < b$ y-value at c for all x in the interval. high point
Relative Minimum lowest point/y-value on an interval	low point any # A function f is said to have a relative minimum at $x = c$ if there is an endpoints are not included: $a < x < b$ or (a,b) open interval containing c on which $f(c)$ is the smallest value $a < c < b$ y-value at c for all x in the interval. low point
Relative Extreme Value highest (max) or lowest (min) values (could be more than 1)	If f has either a relative maximum or a relative minimum at $x = c$, high point low point x-value (where min or max occurs) then f is said to have a relative extreme value at $x = c$. y-value is biggest or smallest
Critical Numbers an x-value where the derivative is zero or does not exist	If c is a number in the domain of the function f, x-values $(x=c)$ and if either $f'(c) = 0$ or x-value the derivative at c is zero y'-value $f'(c)$ does not exist, then the derivative does not exist at $x=c$ (may be divided by zero) c is a critical number of f. $x=c$ is a critical #