October 21

When do you need to create a table to find maximums and minimums (1st derivative test)

relative min/max

CP		
sign f'		
behav f		

and when can you find the value of the function at the critical points and endpoints? (candidate test)

global max/min

$1 \le x \le 5$

$f(1) =$

$f(5) =$

$f(cp) =$

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Students will verbally explain how to use the derivative to give characteristics of a function

(using the words:

increasing, decreasing, positive, negative, zero...)

5. Let $f(x)=\frac{e+\ln x}{x^2}$

(a) Find the average rate of change of f from $x=1$ to $x=e$.

slope - 2 pts

$$\frac{f(b)-f(a)}{b-a}=\frac{f(e)-f(1)}{e-1}=\frac{\left(\frac{e+\ln(e)}{e^2}\right)-\left(\frac{e+\ln(1)}{1^2}\right)}{e-1}=\frac{\left(\frac{e+1}{e^2}\right)-e}{e-1}$$

$=-1.289$

$y-y_1=m(x-x_1)$ $\quad f'(x)$

(b) Write an equation of the line tangent to f at $x=1$.

$$f(x)=\frac{e+\ln x}{x^2} \qquad f'(x)=\frac{(0+\frac{1}{x})(x^2)-2x(e+\ln x)}{(x^2)^2}$$

$$f(1)=\frac{e+\ln(1)}{1^2}=e \qquad f'(1)=\frac{\frac{1}{1}(1^2)-2(1)(e+\ln(1))}{1^4}=\frac{1-2e}{1}=1-2e=-4.437$$

$$y-e=(1-2e)(x-1) \qquad y-2.718=-4.437(x-1)$$

(c) Find the x-coordinate of the point on f at which the tangent line to f is horizontal.

$f'(x)$ $\quad f'(x)=0$

$$f'(x)=\frac{(0+\frac{1}{x})x^2-2x(e+\ln x)}{(x^2)^2}=0$$

top $=0$

$(0+\frac{1}{x})x^2-2x(e+\ln x)=0$

$x-2x(e)-2x(\ln x)=0$

$x(1-2e-2\ln x)=0$

$\cancel{x=0}$ doesn't work b/c dividing by zero

$1-2e-2\ln x=0$

$-4.437-2\ln x=0$

$-2\ln x=4.437$

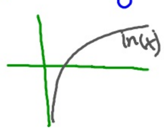
$\ln x=\frac{4.437}{-2}$

$e^{\ln x}=e^{\frac{4.437}{-2}}$

$x=e^{\frac{4.437}{-2}}=.10879$

(d) Find $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to\infty} f(x)$.

$$\lim_{x\to 0^+}\frac{e+\ln x}{x^2}\Rightarrow\frac{e+\ln(0)}{0^2}\Rightarrow\frac{e-\infty}{0}=-\infty$$



$$\lim_{x\to\infty}\frac{e+\ln x}{x^2}\Rightarrow\lim_{x\to\infty}\frac{\ln x}{x^2}\Rightarrow\frac{\text{big \#}}{\text{super big \#}}=0$$

3. If $f(x)=(2x+1)^4$, then the 4th derivative of $f(x)$ at $x=0$ is

(A) 0 (B) 24 (C) 48 (D) 240 (E) 384

$$f'(x)=4(2x+1)^3(2)=8(2x+1)^3$$

$$f''(x)=24(2x+1)^2(2)=48(2x+1)^2$$

$$f'''(x)=96(2x+1)(2)=192(2x+1)$$

$$f^{(4)}(x)=192(2)=384$$

4. If $y=\dfrac{3}{4+x^2}$, then $\dfrac{dy}{dx}=$

(A) $\dfrac{-6x}{\left(4+x^2\right)^2}$ (B) $\dfrac{3x}{\left(4+x^2\right)^2}$ (C) $\dfrac{6x}{\left(4+x^2\right)^2}$ (D) $\dfrac{-3}{\left(4+x^2\right)^2}$ (E) $\dfrac{3}{2x}$

$$\frac{0(4+x^2)-2x(3)}{(4+x^2)^2}$$

6. If $f(x)=x$, then $f'(5)=$

(A) 0 (B) $\dfrac{1}{5}$ (C) 1 (D) 5 (E) $\dfrac{25}{2}$

$$f'(x)=1$$

7. The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

$$y' = \frac{1}{\frac{x}{2}}\left(\frac{1}{2}\right)$$

$$\frac{1}{\frac{4}{2}}\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$

$$\frac{1}{1}\cdot\frac{2}{x}\left(\frac{1}{2}\right) = \frac{2}{2x}$$

$$\frac{x}{2} = \frac{1}{2}x$$