October 28

What information about the function can you learn from the second derivative?

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Students will verbally explain how to use the derivative to give characteristics of a function

(using the words:
concavity, increasing, decreasing, positive, negative, zero...)

Use concavity to determine if each critical point is a minimum or maximum
$y = 6x^5 - 10x^3$

$f'(x) = 0$

$y' = 30x^4 - 30x^2$

$0 = 30x^4 - 30x^2$

$0 = 30x^2(x^2 - 1)$

$\frac{0}{30} = \frac{30x^2}{30}$ $\qquad 0 = x^2 - 1 = (x-1)(x+1)$

$\sqrt{0} = \sqrt{x^2}$ $\qquad +1 \quad +1$

$0 = x$ $\qquad \sqrt{1} = \sqrt{x^2}$

$\qquad -1, 1 = x$

critical points

(plug into second derivative)

$f''(x)$

- if $f''(x) > 0$, $f(x)$ is concave up
- if $f''(x) < 0$, $f(x)$ is concave down

$y' = 30x^4 - 30x^2$

$y'' = 120x^3 - 60x$

$y''(1) = 120(1)^3 - 60(1) = 60$

$\Rightarrow$ positive $\Rightarrow$ concave up

minimum at $x=1$ because $y''(1) > 0$ so y is concave up.

$y''(-1) = 120(-1)^3 - 60(-1) = -60$

$\Rightarrow$ negative $\Rightarrow$ concave down

maximum at $x=-1$ because $y''(-1) < 0$ so y is concave down

$y''(0) = 120(0)^3 - 60(0) = 0$

not enough information to determine min or max

2nd Derivative Test for Minimums and Maximums

1. find critical points ($f'(x) = 0$ or undefined)
2. evaluate $f''(x)$ at critical pts (plug in crit. pts)
3. Write statements:
   - max at $x =$ ___ because $f''(\_) < 0$ so $f(x)$ is concave down
   - min at $x =$ ___ because $f''(\_) > 0$ so $f(x)$ is concave up