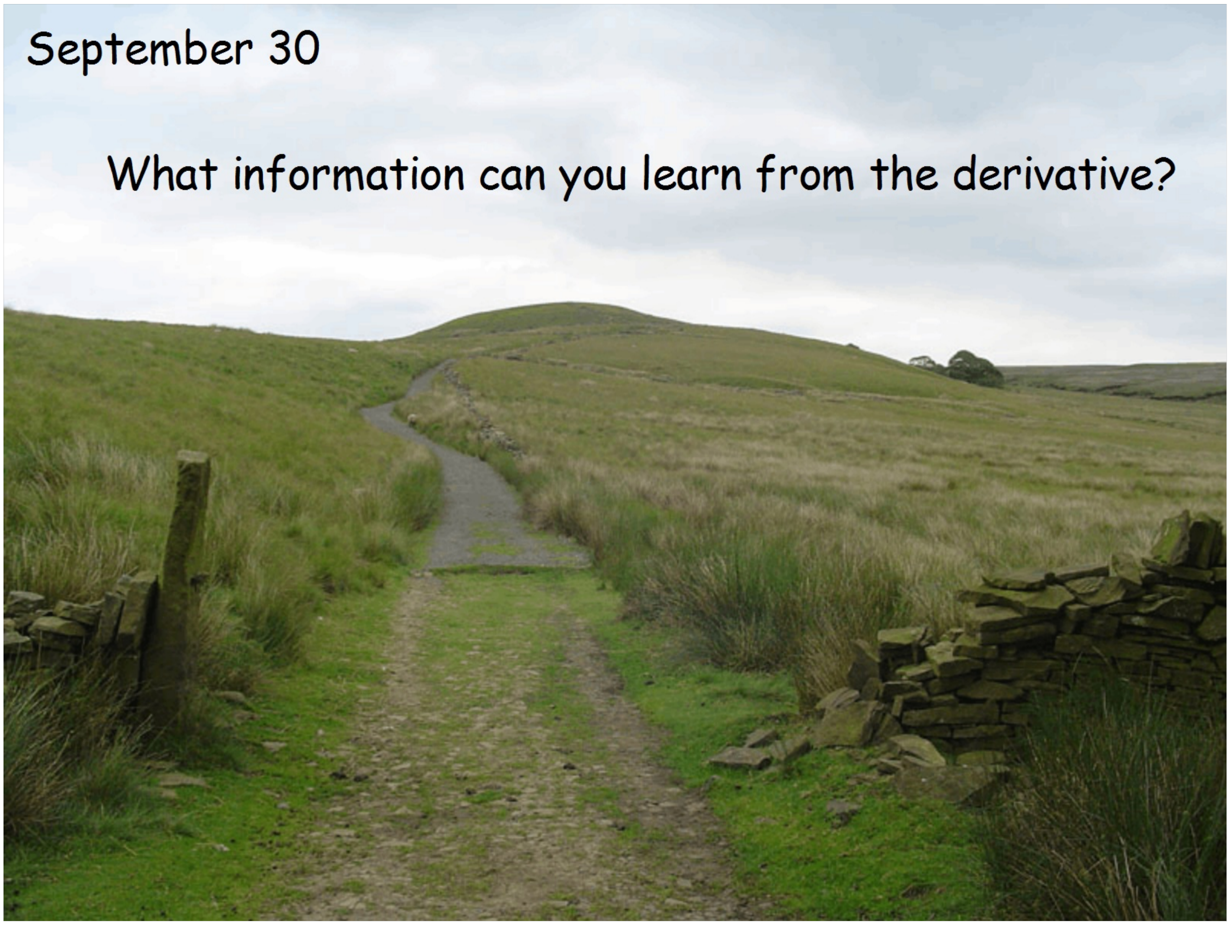

September 30

Students will verbally explain how to

find the derivative

(using the words:

function, exponent, coefficient...)

☺ Derivatives - Power Rule, Product Rule, Quotient Rule, Chain Rule
(half worksheets, worksheet from book, step-by-step problems)

☺ Definition of the Derivative
(worksheet)

☺ Equation of a tangent line
(on all of above worksheets)

8. AB Calculus – Step-by-Step **Name** ____________________

Parts a, b, and c all refer to $f(x)$, given by $f(x)=x^2-x-6$ which is defined on [0, 6].

$y-y_1=m(x-x_1)$

a. Write an equation of the line tangent to f at the point where $x=4$.

$f'(x)$ $f'(4)$

$$f(x)=x^2-x-6$$

$$f(4)=4^2-4-6=6 \rightarrow\rightarrow (4,6)$$

Point

y - 6 = 7(x - 4)

$$f'(x)=2x-1$$

$$f'(4)=2(4)-1=7 \rightarrow \text{slope}$$

b. If $g(x)=[f(x)]^2$,write an equation of any horizontal tangent lines to g. Show how you arrive at your answer.

$y-y_1=m(x-x_1)$ $g'(x)=0$

$$g(x)=[f(x)]^2=[x^2-x-6]^2$$

$$g'(x)=2f(x)\cdot f'(x)$$

$$g'(x)=2(x^2-x-6)(2x-1)$$

$$0=2(x^2-x-6)(2x-1)$$

$$0=2(x-3)(x+2)(2x-1)$$

$0\neq 2$ $\quad 0=x-3$ $\quad 0=x+2$ $\quad 0=2x-1$

$x=3$ $\quad x=-2$ $\quad x=\frac{1}{2}$

$$g(3)=(3^2-3-6)^2=(9-3-6)^2=0$$

$$g(-2)=((-2)^2-(-2)-6)^2=(4+2-6)^2=0$$

$$g\left(\tfrac{1}{2}\right)=\left(\left(\tfrac{1}{2}\right)^2-\tfrac{1}{2}-6\right)^2=\left(\tfrac{1}{4}-\tfrac{2}{4}-\tfrac{24}{4}\right)^2=\left(-\tfrac{25}{4}\right)^2$$

$\boxed{y=0}$

$$y=\left(-\frac{25}{4}\right)^2$$

$$y-\left(\frac{-25}{4}\right)^2=0\left(x-\frac{1}{2}\right)$$

$$y-0=0(x+2)$$

c. If $h(x)=\dfrac{1}{f(2x)}$, find all values of x where the tangent lines to h are either horizontal or do not exist on the interval [0, 6]. Show how you arrive at your answer.

$h'(x)$ $\quad h'(x)=0$ $\quad h'(x)=\frac{\#}{0}$

$$h(x)=\frac{1}{f(2x)}=\frac{1}{(2x)^2-(2x)-6}=\frac{1}{4x^2-2x-6}$$

$$h'(x)=\frac{0(4x^2-2x-6)-(8x-2)(1)}{(4x^2-2x-6)^2}=\frac{-(8x-2)}{(4x^2-2x-6)^2}$$

$f(x)=x^2-x-6$

$f(2x)=(2x)^2-(2x)-6$

(top = 0)
horizontal

$$0=-(8x-2)$$

$$0=8x-2$$

$$\boxed{x=\tfrac{2}{8}=\tfrac{1}{4}}$$

(bottom = 0)
undefined

$$0=4x^2-2x-6$$

$$0=2(2x^2-x-3)$$

$$0=2(2x-3)(x+1)$$

$0\neq 2$ $\quad 0=2x-3$ (+3, +3) $\quad 0=x+1$

$\boxed{x=\tfrac{3}{2}}$ $\quad \boxed{x=-1}$

4. AB Calculus – Step-by-Step Name ______________________________

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	-3	5	2
2	-3	-1	4	6
3	π	8	-1	4
4	-5	Unknown	0	3

The functions f and g are differentiable for all real numbers g. The table above gives values of the function and their first derivatives at selected values of x.

a. If the function h is given by $h(x)=\dfrac{f(x)}{g(x)}+x$, find $h'(1)$.

b. If the function r is given by $r(x)=-2f(x)g(x)$, find the equation of the tangent line to $r(x)$ at $x=2$.

c. If the function v is given by $v(x)=\dfrac{f(x)-1}{f(x)}$, find the slope of the line parallel to v at $x=3$.

(same slope)

d. If the function w is given by $w(x)=xf(x)$ and $w'(4)=9$, find $f'(4)$.

4. AB Calculus – Step-by-Step Name ______________________________

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The functions f and g are differentiable for all real numbers g. The table above gives values of the function and their first derivatives at selected values of x.

a. If the function h is given by $h(x)=\dfrac{f(x)}{g(x)}+x$, find $h'(1)$.

$$h(x)=\frac{f(x)}{g(x)}+x$$

$$h'(x)=\frac{f'(x)g(x)-g'(x)f(x)}{g(x)^2}+1$$

$$h'(1)=\frac{f'(1)g(1)-g'(1)f(1)}{g(1)^2}+1=\frac{-3(5)-2(4)}{(5)^2}+1=\frac{-15-8}{25}+1=\frac{-23}{25}+\frac{25}{25}=\frac{2}{25}$$

b. If the function r is given by $r(x)=-2f(x)g(x)$, find the equation of the tangent line to $r(x)$ at $x=2$.

$$r(x)=-2f(x)g(x)$$

$$r'(x)=-2f'(x)g(x)+g'(x)(-2f(x))$$

$$r'(2)=-2f'(2)g(2)+g'(2)(-2f(2))=-2(-1)(4)+6(-2)(-3)=8+36=44$$

$$r(2)=-2f(2)g(2)=-2(-3)(4)=24$$

$y-24=44(x-2)$

c. If the function v is given by $v(x)=\dfrac{f(x)-1}{f(x)}$, find the slope of the line parallel to v at $x=3$.

$$v(x)=\frac{f(x)-1}{f(x)}$$

$$v'(x)=\frac{f'(x)\cdot f(x)-f(x)(f(x)-1)}{f(x)^2}$$

$$v'(3)=\frac{f'(3)\cdot f(3)-f'(3)(f(3)-1)}{f(3)^2}=\frac{8(\pi)-8(\pi-1)}{\pi^2}=\frac{8\pi-8\pi+8}{\pi^2}=\frac{8}{\pi^2}$$

d. If the function w is given by $w(x)=xf(x)$ and $w'(4)=9$, find $f'(4)$.

$$w(x)=xf(x)$$

$$w'(x)=1f(x)+f'(x)x$$

$$w'(4)=1f(4)+f'(4)4=9$$

$$f(4)+4f'(4)=9$$

$$-5+4f'(4)=9$$

$$4f'(4)=14$$

$$f'(4)=\tfrac{14}{4}$$

$$f'(4)=\tfrac{7}{2}$$