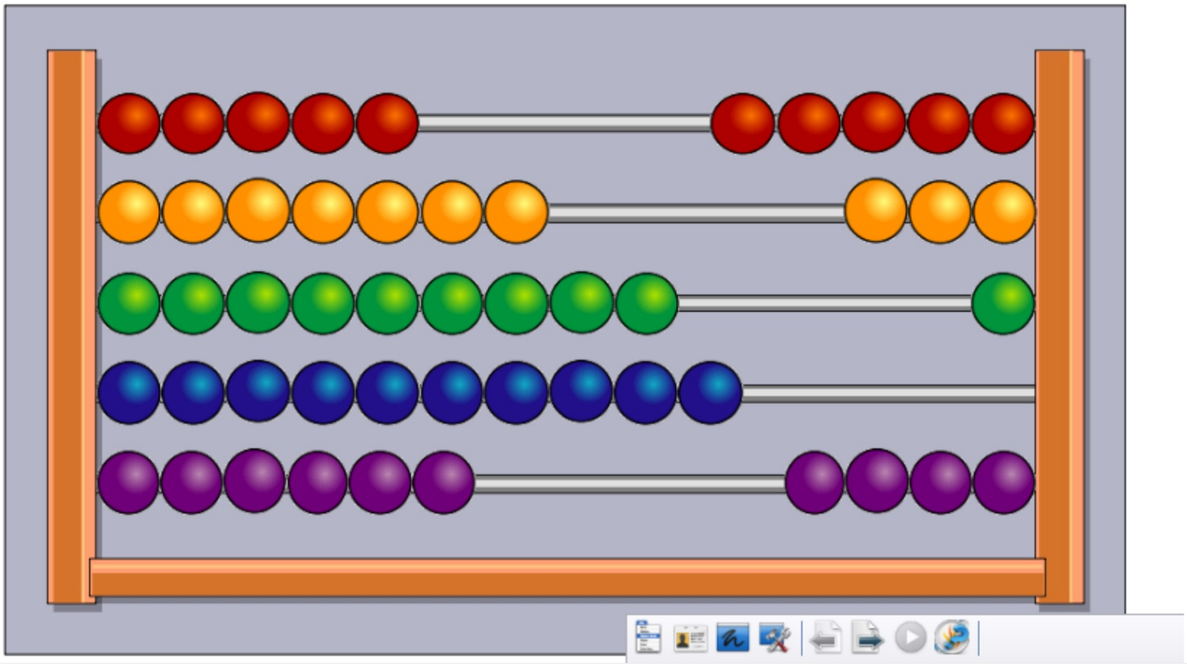
SWBAT:

Solve Problems with Calculus



A - Horizontal Asymptotes
D - Equation of a Tangent Line
L - 2^{nd} Derivative Test
N - Computation of Riemann Sums
O - Accumulation Function (FTC)
Q - Derivative of Accumulation Function (FTC - part 2)
A - L'Hopital's Rule
C - Improper Integrals
H - Vector Valued Functions
N - Power Series

Let f be a function with derivatives of all orders and for which $f(2) = 7$. When n is odd, the nth derivative of f at $x = 2$ is 0. When n is even and $n \geq 2$, the nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \dfrac{(n-1)!}{3^n}$.

(a) Write the sixth-degree Taylor polynomial for f about $x = 2$.

(b) In the Taylor series for f about $x = 2$, what is the coefficient of $(x-2)^{2n}$ for $n \geq 1$?

(c) Find the interval of convergence of the Taylor series for f about $x = 2$. Show the work that leads to your answer.

Taylor Polynomial: $\sum_{n=0}^{6} f^{(n)}(2)\cdot\frac{(x-2)^n}{n!}$

$$\sum_{n=0}^{?} \frac{f^{(n)}(a)(x-a)^n}{n!}$$

$f(2)=7$ $\quad f'''(2)=0$ $\quad f^{(5)}(2)=0$

$f'(2)=0$ $\quad f^{(4)}(2)=\frac{3!}{3^4}$ $\quad f^{(6)}(2)=\frac{5!}{3^6}$

$f''(2)=\frac{1}{9}$

$$P(x) = 7 + \frac{0(x-2)}{1!} + \frac{1}{9}\frac{(x-2)^2}{2!} + 0 + \frac{3!}{3^4}\frac{(x-2)^4}{4!} + 0 + \frac{5!}{3^6}\frac{(x-2)^6}{6!}$$

$$= 7 + \frac{1}{3^2}\frac{(x-2)^2}{2!} + \frac{3!}{3^4}\frac{(x-2)^4}{4!} + \frac{5!}{3^6}\frac{(x-2)^6}{6!}$$

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$$7 + \underbrace{\frac{1}{3^2}\cdot\frac{(x-2)^{2\cdot 1}}{2!}}_{(n=1)} + \underbrace{\frac{3!}{3^4}\cdot\frac{(x-2)^{2\cdot 2}}{4!}}_{(n=2)} + \underbrace{\frac{5!}{3^6}\cdot\frac{(x-2)^{2\cdot 3}}{6!}}_{(n=3)}$$

$$\frac{(2n-1)!}{3^{2n}(2n)!}(x-2)^{2n}$$

$$7 + \frac{1}{3^2(2)!}(x-2)^2 + \frac{3!}{3^4(4)!}(x-2)^4$$

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$$= 7 + \frac{1}{3^2}\frac{(x-2)^2}{2!} + \frac{3!}{3^4}\frac{(x-2)^4}{4!} + \frac{5!}{3^6}\frac{(x-2)^6}{6!} + \cdots + \frac{(2n-1)!}{3^{2n}} \cdot \frac{(x-2)^{2n}}{(2n)!}$$

$$= 7 + \sum_{n=1}^{\infty} \frac{(2n-1)!}{3^{2n}} \cdot \frac{(x-2)^{2n}}{(2n)!}$$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(2(n+1)-1)!}{3^{2(n+1)}}\cdot\frac{(x-2)^{2(n+1)}}{(2(n+1))!}}{\frac{(2n-1)!}{3^{2n}}\cdot\frac{(x-2)^{2n}}{(2n)!}}\right| = \lim_{n\to\infty}\left|\frac{(x-2)^2(2n)}{3^2(2(n+1))}\right| = \frac{(x-2)^2}{3^2}$$

Converges when

$$-1 < \frac{(x-2)^2}{3^2} < 1$$

$$-1 < \frac{x-2}{3} < 1$$

$$-3 < x-2 < 3$$

$$-1 < x < 5$$

at $x=-1$

$$\sum \frac{(2n-1)!}{3^{2n}}\cdot\frac{(-1-2)^{2n}}{(2n)!} = \sum \frac{1}{2n}\frac{(-3)^{2n}}{(3)^{2n}} = \sum \frac{1}{2n}\left(\frac{-3}{3}\right)^{2n} = \sum \frac{1}{2n}(-1)^{2n} = \sum \frac{1}{2n}$$

Harmonic
$\Rightarrow$ Diverges

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Question 6

Let f be a function with derivatives of all orders and for which $f(2) = 7$. When n is odd, the nth derivative of f at $x = 2$ is 0. When n is even and $n \ge 2$, the nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \frac{(n-1)!}{3^n}$.

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(a) $P_6(x) = 7 + \frac{1!}{3^2}\cdot\frac{1}{2!}(x-2)^2 + \frac{3!}{3^4}\cdot\frac{1}{4!}(x-2)^4 + \frac{5!}{3^6}\cdot\frac{1}{6!}(x-2)^6$

$f(x) = P_6(x)$

$f(x) \approx$

3 : { 1 : polynomial about $x = 2$; 2 : $P_6(x)$; $\langle -1\rangle$ each incorrect term; $\langle -1\rangle$ max for all extra terms, $+\cdots$, misuse of equality }

(b) $\frac{(2n-1)!}{3^{2n}}\cdot\frac{1}{(2n)!} = \frac{1}{3^{2n}(2n)}$

1 : coefficient

(c) The Taylor series for f about $x = 2$ is

$$f(x) = 7 + \sum_{n=1}^{\infty}\frac{1}{2n\cdot 3^{2n}}(x-2)^{2n}.$$

$$L = \lim_{n\to\infty}\left|\frac{\frac{1}{2(n+1)}\cdot\frac{1}{3^{2(n+1)}}(x-2)^{2(n+1)}}{\frac{1}{2n}\cdot\frac{1}{3^{2n}}(x-2)^{2n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{2n}{2(n+1)}\cdot\frac{3^{2n}}{3^2 3^{2n}}(x-2)^2\right| = \frac{(x-2)^2}{9}$$

$L < 1$ when $|x-2| < 3$.

Thus, the series converges when $-1 < x < 5$.

When $x = 5$, the series is $7 + \sum_{n=1}^{\infty}\frac{3^{2n}}{2n\cdot 3^{2n}} = 7 + \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$,

which diverges, because $\sum_{n=1}^{\infty}\frac{1}{n}$, the harmonic series, diverges.

When $x = -1$, the series is $7 + \sum_{n=1}^{\infty}\frac{(-3)^{2n}}{2n\cdot 3^{2n}} = 7 + \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$,

which diverges, because $\sum_{n=1}^{\infty}\frac{1}{n}$, the harmonic series, diverges.

The interval of convergence is $(-1, 5)$.

5 : { 1 : sets up ratio; 1 : computes limit of ratio; 1 : identifies interior of interval of convergence; 1 : considers both endpoints; 1 : analysis/conclusion for both endpoints }