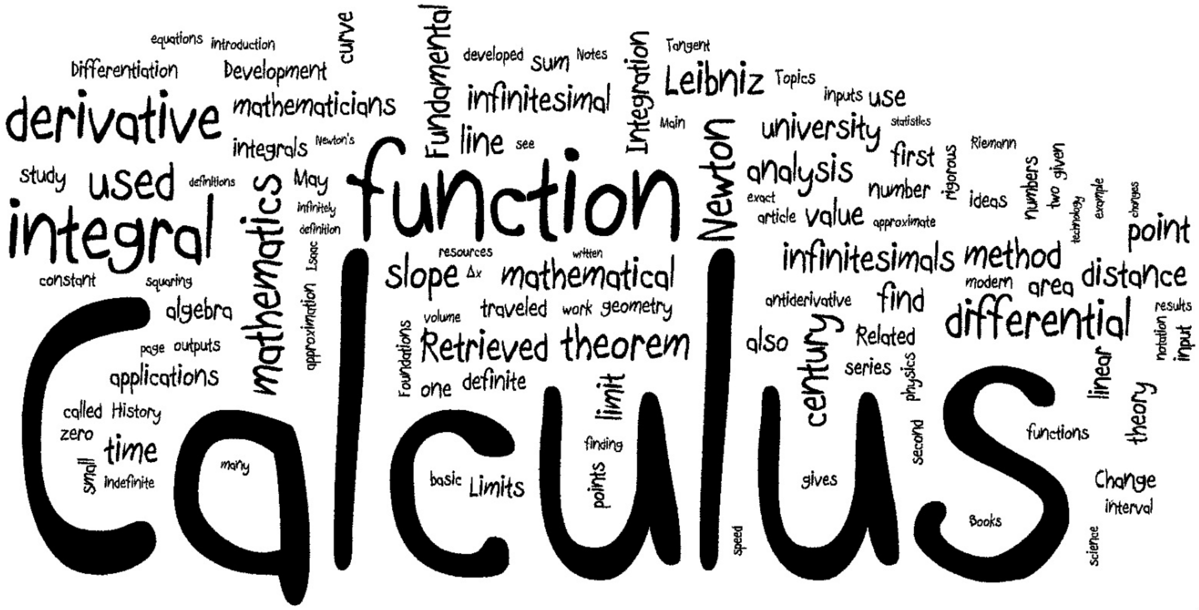


Tips for area and volume problems:

- Draw the graph (if it is not drawn for you)

- Area = $\int$ top function - bottom function dx

top or bottom could be a line
[y = 2, y = 0 (the x-axis)]

$\int$ right function - left function dy

- Volumes of Revolution

~ Draw in the radius

~ vertical line → y =...
~ horizontal line → x =...

~ if you are rotating around a line other than the x- (or y-) axis it is part of both radii

$(UF - line)^2 - (LF - line)^2$

$$\text{Volume} = \pi \int (\text{Outside Radius})^2 - (\text{Inside Radius})^2$$

- Volumes by slicing

* you are given the shape of the cross-section

* Draw in the slice

  * vertical line → y =...
  * horizontal line → x =...

* One dimension (side, diameter, etc) is the distance between the functions (top - bottom)

$$* \text{ Volume} = \int \text{Area of shape}$$

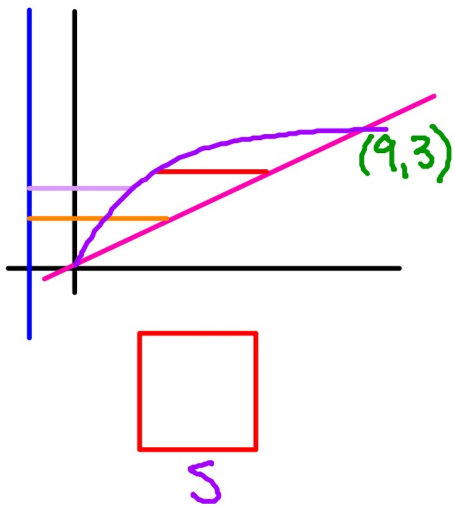
2008 AB/BC 1

1. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \dfrac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.



(a) $\displaystyle\int_0^9 \sqrt{x} - \frac{x}{3}\,dx = 4.5$

(b) $y = \sqrt{x} \rightarrow x = y^2$

$y = \frac{x}{3} \rightarrow x = 3y$

$OR = 3y - (-1) = 3y + 1$

$IR = y^2 - (-1) = y^2 + 1$

$\displaystyle V = \pi\int_0^3 (3y+1)^2 - (y^2+1)^2\,dy$

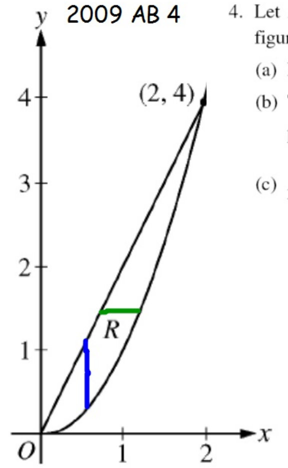
$= 130.061$

$= 41.4\pi$

(c) $s = 3y - y^2 \quad A = (3y - y^2)^2$

$$\int_0^3 (3y - y^2)^2\,dy = 8.1$$

2009 AB 4



4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

(a) $\displaystyle\int_0^2 2x - x^2\,dx = x^2 - \frac{x^3}{3}\Big|_0^2$

$= 2^2 - \frac{2^3}{3} - \left(0^2 - \frac{0^3}{3}\right)$

$= 4 - \frac{8}{3} = \frac{4}{3}$

(b) $\displaystyle\int_0^2 \sin\left(\frac{\pi}{2}x\right)dx = -\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Big|_0^2$

$= -\frac{2}{\pi}\cos\left(\frac{\pi}{2}(2)\right) - \left(-\frac{2}{\pi}\cos\left(\frac{\pi}{2}(0)\right)\right)$

$= -\frac{2}{\pi}(-1) - \left(-\frac{2}{\pi}\right) = \frac{2}{\pi} + \frac{2}{\pi} = \frac{4}{\pi}$

$u = \frac{\pi}{2}x$ $\quad \int \sin(u)\left(\frac{2}{\pi}du\right)$

$du = \frac{\pi}{2}dx$ $\quad \frac{2}{\pi}\int \sin(u)\,du$

$\frac{2}{\pi}du = dx$ $\quad -\frac{2}{\pi}\cos(u) + C$

(c)

$y = 2x \rightarrow x = \frac{y}{2}$

$y = x^2 \rightarrow x = \sqrt{y}$

$s = \sqrt{y} - \frac{y}{2}$

$$V = \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$$