

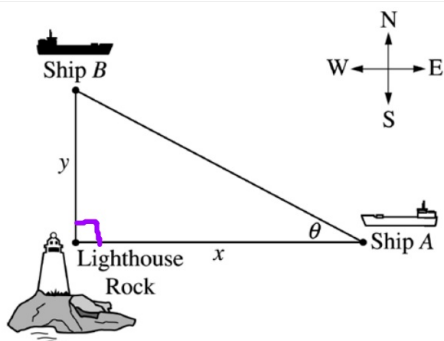


SWBAT:

# SOLVE RELATED RATE PROBLEMS

- (1) State what you know and what you are looking for \*
- (2) take the differential of each term
- (3) divide by  $dt$
- (4) substitute values you have
- (5) solve

\* sometimes you may have to solve for a variable on first



$$x = 4 \quad y = 3 \quad h = 5$$

$$\frac{dx}{dt} = -15 \quad \frac{dy}{dt} = 10 \quad \frac{dh}{dt} = ?$$

6. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let  $x$  be the distance between Ship A and Lighthouse Rock at time  $t$ , and let  $y$  be the distance between Ship B and Lighthouse Rock at time  $t$ , as shown in the figure above.

(a) Find the distance, in kilometers, between Ship A and Ship B when  $x = 4$  km and  $y = 3$  km.

$$x^2 + y^2 = h^2 \rightarrow 3^2 + 4^2 = h^2 \quad h = 5 \text{ km}$$

(b) Find the rate of change, in km/hr, of the distance between the two ships when  $x = 4$  km and  $y = 3$  km.

$$x^2 + y^2 = h^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2h \frac{dh}{dt}$$

$$\frac{2(4)(-15) + 2(3)(10)}{2(5)} = \frac{dh}{dt} = -6 \text{ km/hr}$$

(c) Let  $\theta$  be the angle shown in the figure. Find the rate of change of  $\theta$ , in radians per hour, when  $x = 4$  km and  $y = 3$  km.

$$\frac{d\theta}{dt} = ? \quad \tan \theta = \frac{y}{x}$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{x \left( \frac{dy}{dt} \right) - y \left( \frac{dx}{dt} \right)}{x^2}$$

$$\sec^2 \left( \tan^{-1} \left( \frac{3}{4} \right) \right) \frac{d\theta}{dt} = \frac{4(10) - 3(-15)}{4^2}$$

$$\frac{d\theta}{dt} = \frac{85}{16} \cos^2 \left( \tan^{-1} \left( \frac{3}{4} \right) \right)$$

$$\tan \theta = \frac{3}{4}$$

$$\theta = \tan^{-1} \left( \frac{3}{4} \right)$$