August 27

Students will verbally explain how to
use derivatives to analyze graphs
(using appropriate vocabulary:
sign change, concave up, concave down, slope, point)
by solving an AP Problem in groups

What do you need to write an equation of a tangent line?	point slope = derivative
What is an inflection point? How do you find one?	Where the sign of the 2nd derivative changes (Where the concavity changes)
What is the second derivative test for extrema?	($x=a$ is a critical point) if $f''(a) < 0$, $f(x)$ is concave down + has a max at $x=a$ if $f''(a) > 0$, $f(x)$ is concave up + has a min at $x=a$

4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

(a) Write an equation of the line tangent to the graph of $\boxed{f}$ at the point where $x = 0$.

$y - 2 = -3x$

$y = -3x + 2$

(a) Slope at $x = 0$ is $f'(0) = -3$ At $x = 0$, $y = 2$ $y - 2 = -3(x - 0)$	1: equation

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

[redacted]

(b) No. Whether $f''(x)$ changes sign at $x = 0$ is unknown. The only given value of $f''(x)$ is $f''(0) = 0$.	**2** $\begin{cases} \text{1: answer} \\ \text{1: explanation} \end{cases}$

Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of $\boxed{g}$ at the point where $x = 0$.

$g'(0) = e^{-2(0)}(3f(0) + 2f'(0))$

[redacted]

$g'(0) = e^{0}(3(2) + 2(-3)) = 0$

(c) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ $g'(0) = e^{0}(3f(0) + 2f'(0))$ $= 3(2) + 2(-3) = 0$ $y - 4 = 0(x - 0)$ $y = 4$	**2** $\begin{cases} \text{1: } g'(0) \\ \text{1: equation} \end{cases}$

(d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

(d) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ $g''(x) = (-2e^{-2x})(3f(x) + 2f'(x))$ $+ e^{-2x}(3f'(x) + 2f''(x))$ $= e^{-2x}(-6f(x) - f'(x) + 2f''(x))$ $g''(0) = e^{0}[(-6)(2) - (-3) + 2(0)] = -9$ Since $g'(0) = 0$ and $g''(0) < 0$, g does have a local maximum at $x = 0$.	**4** $\begin{cases} \text{2: verify derivative} \\ \quad \text{0/2 product or chain rule error} \\ \quad \text{<-1> algebra errors} \\ \text{1: } g'(0) = 0 \text{ and } g''(0) \\ \text{1: answer and reasoning} \end{cases}$