BC Calculus
Fall Review

Get Started!

Jeopardy

Derivatives	Applications of Derivatives	Integrals	Applications of Integrals	Extra Topics
100	100	100	100	~~100~~
200	200	200	200	~~200~~
~~300~~	300	~~300~~	300	300
400	~~400~~	~~400~~	400	~~400~~
~~500~~	500	~~500~~	500	~~500~~

Team 1 1600

Team 2 1100

Take me to Final Jeopardy

Derivatives

If $f(x) = \sqrt{4\sin x + 2}$, then $f'(0) =$

(A) -2 (B) 0 (C) 1 $\boxed{\text{(D) } \frac{\sqrt{2}}{2}}$ $\boxed{\text{(E) } \sqrt{2}}$

Derivatives

(E) $\sqrt{2}$

$$f(x) = \sqrt{4\sin x + 2}$$

$$f(x) = (4\sin x + 2)^{\frac{1}{2}}$$

$$f'(x) = \tfrac{1}{2}(4\sin x + 2)^{-\frac{1}{2}}(4\cos x)$$

$$f'(x) = \frac{4\cos x}{2\sqrt{4\sin x + 2}}$$

$$f'(0) = \frac{4\cos 0}{2\sqrt{4\sin 0 + 2}} = \frac{4(1)}{2\sqrt{4(0) + 2}}$$

$$= \frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

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Derivatives

If $\frac{dy}{dx} = x^2y^2$, then $\frac{d^2y}{dx^2} =$

(A) $2xy^2$

(B) $4x^3y^3$

(C) $2x + 2x^2y^3$

(D) $2x^2y + 2xy^2$

(E) $2x^4y^3 + 2xy^2$

Derivatives

(E) $2x^4y^3 + 2xy^2$

$$\frac{dy}{dx} = x^2y^2$$

$$\frac{d^2y}{dx^2} = \left(2x\frac{dx}{dx}\right)\left(y^2\right) + \left(2y\frac{dy}{dx}\right)\left(x^2\right) =$$

$$\left(2x\right)\left(y^2\right) + \left(2y\left(x^2y^2\right)\right)\left(x^2\right) =$$

$$2xy^2 + 2x^4y^3$$

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Applications of Derivatives

If the graph of $f(x) = 2x^2 + \frac{k}{x}$ has a point of inflection at $x = -1$, then the value of k is

(A) -2 $\boxed{\text{(B) -1}}$ (C) 0 (D) 1 $\boxed{\text{(E) 2}}$

Applications of Derivatives

(E) 2

$$f(x) = 2x^2 + \frac{k}{x} = 2x^2 + kx^{-1}$$

$$f'(x) = 4x - kx^{-2}$$

$$f''(x) = 4 + 2kx^{-3} = 4 + \frac{2k}{x^3}$$

inflection point at $x = -1$

$$0 = 4 + \frac{2k}{(-1)^3} = 4 - 2k$$

$$4 = 2k \Rightarrow k = 2$$

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Integrals

$$4\int_{1}^{e^2} \frac{x - x^3}{x^2}\,dx =$$

(A) $3 - e^2$

(B) $3 - e^4$

$\boxed{\text{(C) } 5 - e^2}$

(D) $5 - e^4$

$\boxed{\text{(E) } 10 - 2e^4}$

Integrals

(E) $10 - 2e^4$

$$4\int_{1}^{e^2} \frac{x - x^3}{x^2}\,dx = 4\int_{1}^{e^2} \frac{x}{x^2} - \frac{x^3}{x^2}\,dx = 4\int_{1}^{e^2} \frac{1}{x} - x\,dx =$$

$$4\left(\ln x - \frac{x^2}{2}\Big|_{1}^{e^2}\right) = 4\left[\left(\ln\left(e^2\right) - \frac{\left(e^2\right)^2}{2}\right) - \left(\ln(1) - \frac{1^2}{2}\right)\right] =$$

$$4\left(2 - \frac{e^4}{2} - 0 + \frac{1}{2}\right) = 8 - 2e^2 + 2 = 10 - 2e^2$$

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Integrals

$$\int \frac{1}{\sqrt{4 - x^2}}\, dx =$$

(A) $\text{Arcsin}\,\frac{x}{2} + C$

(B) $2\sqrt{4 - x^2} + C$

(C) $\text{Arcsin}\, x + C$

(D) $\sqrt{4 - x^2} + C$

(E) $\frac{1}{2}\text{Arcsin}\,\frac{x}{2} + C$

Integrals

(A) $\text{Arcsin}\,\frac{x}{2} + C$

$$\int \frac{1}{\sqrt{4-x^2}}\,dx = \int \frac{1}{\sqrt{4\left(1-\frac{x^2}{4}\right)}}\,dx =$$

$$\int \frac{1}{\sqrt{4}}\,\frac{1}{\sqrt{\left(1-\left(\frac{x}{2}\right)^2\right)}}\,dx = \int \frac{1}{2}\,\frac{1}{\sqrt{\left(1-\left(\frac{x}{2}\right)^2\right)}}\,dx$$

$$u = \tfrac{x}{2} \Rightarrow du = \tfrac{1}{2}dx \Rightarrow 2du = dx$$

$$\int \frac{1}{2}\,\frac{1}{\sqrt{\left(1-u^2\right)}}\,2du = \int \frac{1}{\sqrt{\left(1-u^2\right)}}\,du =$$

$$\text{Arcsin}(u) + C = \text{Arcsin}\left(\tfrac{x}{2}\right) + C$$

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Integrals

Let $f(x)$ be the function defined by $f(x) = \begin{cases} x, & x \le 0 \\ x+1, & x > 0 \end{cases}$

The value of $\int_{-2}^{1} x\, f(x)\, dx =$

(A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) $\frac{11}{2}$

Integrals

(D) $\frac{7}{2}$

$$f(x) = \begin{cases} x, & x \le 0 \\ x+1, & x > 0 \end{cases}$$

$$\int_{-2}^{1} x\ f(x)\ dx = \int_{-2}^{0} x\ f(x)\ dx + \int_{0}^{1} x\ f(x)\ dx =$$
$$\int_{-2}^{0} x(x)\ dx + \int_{0}^{1} x(x+1)\ dx =$$
$$\int_{-2}^{0} x^2\ dx + \int_{0}^{1} x^2 + x\ dx =$$

$$\left(\frac{x^3}{3}\Big|_{-2}^{0}\right) + \left(\frac{x^3}{3} + \frac{x^2}{2}\Big|_{0}^{1}\right) = \left(\frac{0^3}{3} - \frac{(-2)^3}{3}\right) + \left(\left(\frac{1^3}{3} + \frac{1^2}{2}\right) - \left(\frac{0^3}{3} + \frac{0^2}{2}\right)\right) =$$
$$\left(0 + \frac{8}{3}\right) + \left(\left(\frac{1}{3} + \frac{1}{2}\right) - (0+0)\right) = \frac{9}{3} + \frac{1}{2} = 3 + \frac{1}{2} = \frac{7}{2}$$

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Extra Topics

If f is a continuous function defined by

$$f(x) = \begin{cases} x^2 + bx, & x \le 5 \\ 5\sin\left(\frac{\pi}{2}x\right), & x > 5 \end{cases} \quad \text{then } b =$$

(A) -6 (B) -5 (C) -4 (D) 4 (E) 5

Extra Topics

(C) -4

$$x^2 + bx = 5\sin\left(\frac{\pi}{2}x\right) \quad \text{at } x = 5$$

$$5^2 + 5b = 5\sin\left(\frac{5\pi}{2}\right)$$

$$25 + 5b = 5$$

$$5b = -20$$

$$b = -4$$

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Extra Topics

$$\lim_{x\to\infty} \frac{10^8 x^5 + 10^6 x^4 + 10^4 x^2}{10^9 x^6 + 10^7 x^5 + 10^5 x^3} =$$

$\boxed{\text{(A) } 0}$ (B) 1 (C) -1 $\boxed{\text{(D) } \frac{1}{10}}$ (E) $\frac{-1}{10}$

Extra Topics

(A) 0

$$\lim_{x\to\infty} \frac{10^8 x^5 + 10^6 x^4 + 10^4 x^2}{10^9 x^6 + 10^7 x^5 + 10^5 x^3}$$

$$\Rightarrow \lim_{x\to\infty} \frac{10^8 x^5}{10^9 x^6} \Rightarrow \lim_{x\to\infty} \frac{1}{10x} = 0$$

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A particls moves along the x-axis in such a way that its position at time t is given by $x(t) = \frac{1 - t}{1 + t}$

What is the acceleration of the particle at time $t = 0$

(A) -4 (B) -2 (C) $-\frac{3}{5}$ (D) 2 (E) 4

(E) 4

$$x(t) = \frac{1-t}{1+t}$$

$$v(t) = x'(t) = \frac{-1(1+t)-1(1-t)}{(1+t)^2} = \frac{-1-t-1+t}{(1+t)^2} = \frac{-2}{(1+t)^2} = -2(1+t)^{-2}$$

$$a(t) = v'(t) = x''(t) = 4(1+t)^{-3}(1) = \frac{4}{(1+t)^3}$$

$$a(0) = \frac{4}{(1+0)^3} = 4$$

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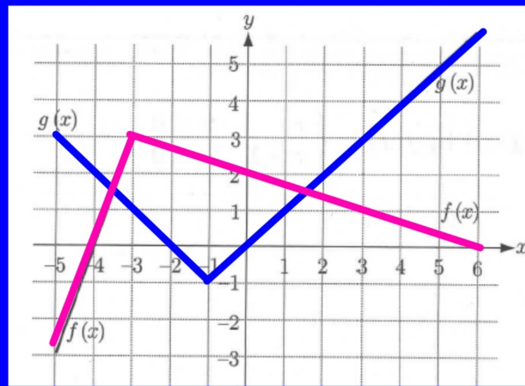
Extra Topics

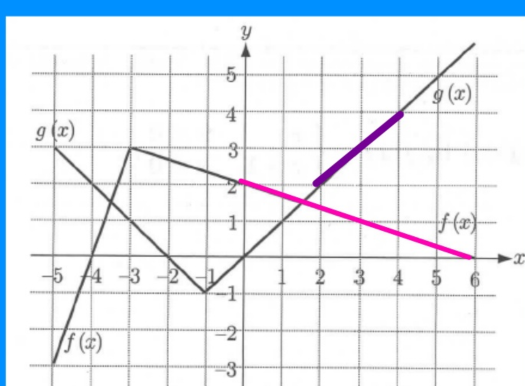


The functions f and g are piecewise linear functions whose graphs are shown above. If $h(x) = f(x)g(x)$, then $h'(3) =$

(A) $-\frac{8}{3}$ (B) $-\frac{1}{3}$ $\boxed{\text{(C) } 0}$ $\boxed{\text{(D) } \frac{2}{3}}$ (E) $\frac{8}{3}$

Extra Topics

(C) 0



$$h(x) = f(x)g(x)$$
$$h'(x) = f'(x)g(x) + g'(x)f(x)$$
$$h'(3) = f'(3)g(3) + g'(3)f(3)$$

$f(3) = 1$ $g(3) = 3$

$f'(3) =$ slope of f at $x = 3$

$f'(3) = -1/3$

$g'(3) =$ slope of g at $x = 3$

$g'(3) = 1$

$$h'(3) = (-1/3)(3) + 1(1)$$
$$= -1 + 1 = 0$$

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