December 2

Add the following two fractions together:

$$\frac{4}{x}+\frac{8}{x+2}=\frac{(x+2)}{(x+2)}\cdot\frac{4}{x}+\frac{x}{x}\cdot\frac{8}{(x+2)}$$

$$=\frac{4x+8+8x}{x(x+2)}=\frac{12x+8}{x(x+2)}$$

December 2

Students will verbally explain how to find the integral using partial fractions

(using the words: decomposition, common denominator...)

$\int \frac{4}{(x-2)(x+1)}\,dx$

$u = x-2 \rightarrow x = u+2$

$du = dx$

$$\int \frac{4}{u(x+1)}\,du = \int \frac{4}{u(u+3)}\,du$$

$$= \int \frac{4}{u^2+3u}\,du = 4\int \frac{1}{u^2+3u}\,du = 4\int (u^2+3u)^{-1}\,du$$

$w = u^2+3u$

$dw = (2u+3)\,du$

$$\int \frac{4}{(x-2)(x+1)}\,dx = \int \frac{A}{x-2} + \frac{B}{x+1}\,dx$$

Decompose the function

separate into a sum of rational functions (with linear denominators)

Decompose $\frac{4}{(x-2)(x+1)}$

$$= \frac{A}{x-2} + \frac{B}{x+1}$$

$$= \frac{(x+1)}{(x+1)} \cdot \frac{A}{(x-2)} + \frac{B}{(x+1)} \cdot \frac{(x-2)}{(x-2)}$$

$$= \frac{A(x+1) + B(x-2)}{(x-2)(x+1)} = \frac{4}{(x-2)(x+1)}$$

$$A(x+1) + B(x-2) = 4$$

let $x = -1$

$$A(-1+1) + B(-1-2) = 4$$

$$0 + -3B = 4 \rightarrow B = -\frac{4}{3}$$

let $x = 2$

$$A(2+1) + B(2-2) = 4$$

$$3A + 0 = 4 \rightarrow A = \frac{4}{3}$$

$$\frac{4}{(x-2)(x+1)} = \frac{\frac{4}{3}}{x-2} + \frac{-\frac{4}{3}}{x+1}$$

Decompose $\dfrac{x+3}{x^2-7x+10}$

$$\frac{x+3}{(x-2)(x-5)} = \frac{A}{x-2} + \frac{B}{x-5} = \frac{-\frac{5}{3}}{x-2} + \frac{\frac{8}{3}}{x-5}$$

$$x+3 = A(x-5) + B(x-2)$$

$x=5$

$$5+3 = 0 + B(5-2) \rightarrow B = \frac{8}{3}$$

$x=2$

$$2+3 = A(2-5) + 0 \rightarrow A = -\frac{5}{3}$$

$$\int \frac{x+3}{x^2-7x+10}\,dx$$

$$= \int \frac{-\frac{5}{3}}{x-2} + \frac{\frac{8}{3}}{x-5}\,dx$$

$$= \int \frac{-\frac{5}{3}}{x-2}\,dx + \int \frac{\frac{8}{3}}{x-5}\,dx = -\frac{5}{3}\int \frac{1}{x-2}\,dx + \frac{8}{3}\int \frac{1}{x-5}\,dx = -\frac{5}{3}\int \frac{1}{u}\,du + \frac{8}{3}\int \frac{1}{u}\,du$$

$u = x-2$, $du = dx$; $u = x-5$, $du = dx$

$$= -\frac{5}{3}\ln|u| + \frac{8}{3}\ln|u| + C$$

$$= \boxed{-\frac{5}{3}\ln|x-2| + \frac{8}{3}\ln|x-5| + C}$$