December 3

When do you need to use partial fractions instead of a u-substitution?

December 3

Students will verbally explain how to find the integral using partial fractions

(using the words:
decomposition, common denominator...)

$$\int \frac{4x}{(2x-3)(x-1)}\,dx$$

$$\int \frac{A}{2x-3} + \frac{B}{x-1}\,dx$$

$$\frac{4x}{\blacksquare(x-1)} \qquad x = \frac{3}{2} \qquad \frac{4\left(\frac{3}{2}\right)}{\left(\frac{3}{2}-1\right)} = \frac{6}{\frac{1}{2}} = 12 = A$$

$$\frac{4x}{(2x-3)\blacksquare} \qquad x = 1 \qquad \frac{4(1)}{(2(1)-3)} = \frac{4}{-1} = -4 = B$$

$$\int \frac{12}{2x-3} + \frac{-4}{x-1}\,dx$$

$$12\int \frac{1}{2x-3}\,dx - 4\int \frac{1}{x-1}\,dx$$

$u = 2x-3$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$12\int \frac{1}{u}\cdot\frac{du}{2} - 4\int \frac{1}{x-1}\,dx$$

$$6\int \frac{1}{u}\,du - 4\int \frac{1}{x-1}\,dx$$

$$6\ln|2x-3| - 4\ln|x-1| + C = \ln\left|\frac{(2x-3)^6}{(x-1)^4}\right| + C$$

Properties of Logs

$$a\ln(b) = \ln(b^a)$$

$$\ln(a) + \ln(b) = \ln(ab)$$

$$\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$$

$$\int \frac{2}{x^3-x}\,dx$$

$$\frac{2}{x^3-x} = \frac{2}{x(x^2-1)} = \frac{2}{x(x-1)(x+1)}$$

$$\int \frac{2}{x^3-x}\,dx = \int \frac{2}{x(x-1)(x+1)}\,dx$$

$$= \int \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}\,dx$$

$$\frac{2}{x(x-1)(x+1)}$$

$x=0 \quad \frac{2}{(0-1)(0+1)} = -2 = A$

$x=1 \quad \frac{2}{1(1+1)} = 1 = B$

$x=-1 \quad \frac{2}{-1(-1-1)} = 1 = C$

$$\int \frac{-2}{x} + \frac{1}{x-1} + \frac{1}{x+1}\,dx$$

$$= -2\int \frac{1}{x}\,dx + \int \frac{1}{x-1}\,dx + \int \frac{1}{x+1}\,dx$$

$$= -2\ln|x| + \ln|x-1| + \ln|x+1| + C$$

$$= \ln\left|x^{-2}(x-1)(x+1)\right| + C$$

$$= \ln\left|\frac{(x-1)(x+1)}{x^2}\right| + C = \ln\left|\frac{x^2-1}{x^2}\right| + C$$