December 4

What are the different types of integration methods you know and when do you use each one?

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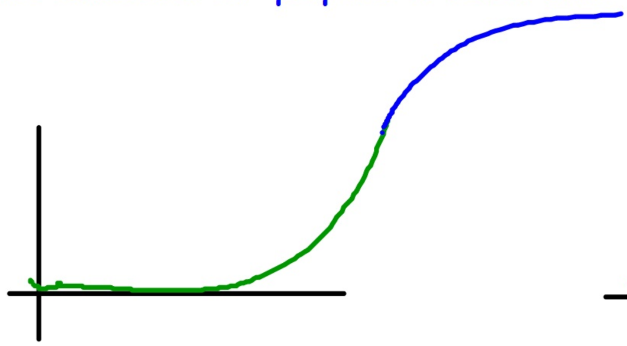
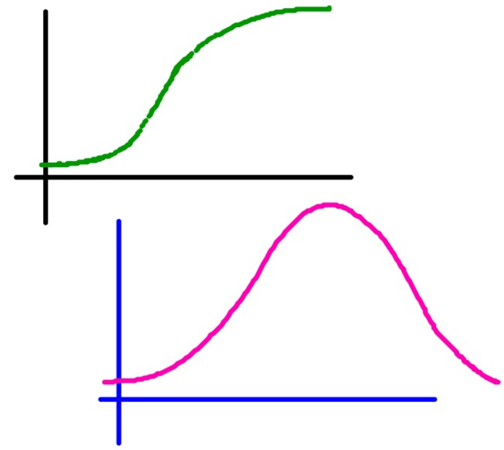
Students will verbally explain how to interpret logistic functions

(using the words:
carrying capacity, rate, constant...)

Microwave Popcorn

Sketch a graph that shows the relationship between the amount of popcorn that has popped and the time





Starts exponential

but has a limiting factor

$$\frac{dP}{dt} = kP(M - P)$$

Logistic Functions

$$\frac{dP}{dt} = KP(M-P)$$

M = maximum (carrying capacity)

$$P = \frac{M}{1+Ae^{-MKt}}$$

K = constant

A = constant that depends on the initial conditions

*rate is a max when $P = \frac{1}{2}M$

The rate at which microwave popcorn pops is modeled by the differential equation:

$$\frac{dP}{dt} = .004P(120 - P)$$

a. What is the carrying capacity for the amount of popcorn in the bag?

120

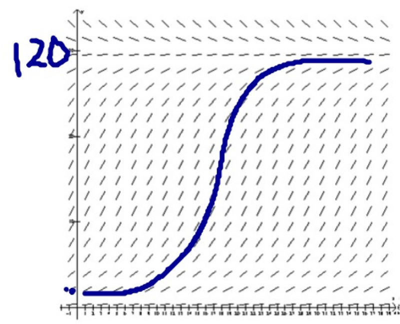
b. What is the total amount of popcorn when it is popping the fastest?

$60 = \frac{120}{2}$

c. What is the rate of change of the amount of popcorn when it is popping the fastest?

$\frac{dP}{dt} = .004(60)(120-60) = 14.4$

d. The slope field is the graph of the given differential equation. If the bag started with 5 popped kernels, sketch a graph of the amount of popcorn over time.



The growth rate of a population of mountain goats in a new game preserve is modeled by the differential equation:

$$\frac{dP}{dt} = 0.008P(30 - P)$$

a. What is the carrying capacity for mountain goats in this game preserve?

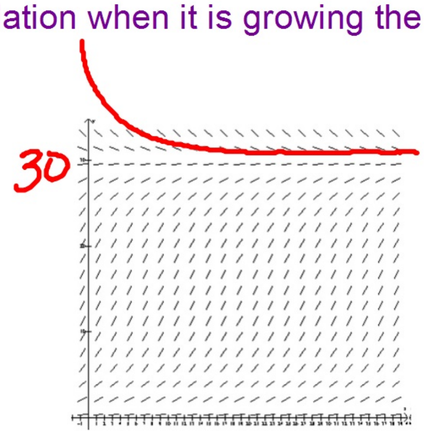
30

b. What is the goat population when the population is growing the fastest?

15

c. What is the rate of change of the goat population when it is growing the fastest?

$1.8 = .008(15)(30-15)$



Logistic Growth - Homework

1. Because of limited food and space, a squirrel population cannot exceed 1200. It grows at a rate proportional both to the existing population and to the attainable additional population.

   a) Write a differential equation that describes this situation.

   $\frac{dP}{dt} = KP(1200-P)$

   b) Write the solution to this differential equation.

   $P = \frac{1200}{1+Ae^{-1200Kt}}$

   c) If there are 100 squirrels two years ago and 400 one year ago, how many squirrels are there now? (Hint - use $\boxed{P(0) = 100}$ and $\boxed{P(1) = 400.}$ You want $P(2)$. Show work.

   $100 = \frac{1200}{1+Ae^{-1200K(0)}}$ ← solve for A

   $400 = \frac{1200}{1+Ae^{-1200K(1)}}$ ← solve for K

   d) Graph the first 5 years of squirrel population.

   e) Use your calculator to find when the squirrel population is growing the fastest.

   $600 = \frac{1200}{1+Ae^{-1200Kt}}$

2. Suppose a flu-like virus is spreading through a population of 50,000 at a rate proportional both to the number of people already infected and to the number still unaffected.

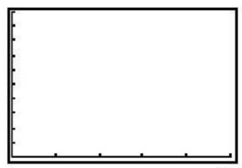
   a) Write a differential equation that describes this situation.

   b) Write the solution to this differential equation.

   c) If 100 people were infected yesterday and 125 are infected today, determine how many will be infected a week from today.

   d) Graph the first 50 days of flu infection.

   e) Use your calculator to find when the flu infection is growing the fastest.



Find a function that models the amount of popcorn

$$\frac{dP}{dt} = .004P(120-P)$$

(1) Separate Variables

$$\frac{1}{P(120-P)}dP = .004dt$$

(2) Integrate both sides (use partial fractions)

$$\int \frac{1}{P(120-P)}dP = \int .004dt$$

$$\int \frac{A}{P}dP + \int \frac{B}{120-P}dP = \int .004dt$$

Find A and B

$$\frac{A}{P} + \frac{B}{120-P} = \frac{1}{P(120-P)}$$

$$A = \frac{1}{120}$$

$$B = \frac{1}{120}$$

$$\int \frac{\frac{1}{120}}{P}dP + \int \frac{\frac{1}{120}}{120-P}dP = \int .004dt$$

$$\tfrac{1}{120}\int \frac{1}{P}dP + \tfrac{1}{120}\int \frac{1}{120-P}dP = \int .004dt$$

$$\tfrac{1}{120}(\ln P) - \tfrac{1}{120}(\ln(120-P)) = .004t + C$$

(3) Simplify

$$\tfrac{1}{120}((\ln P) - (\ln(120-P))) = .004t + C$$

$$120 \cdot \left[\tfrac{1}{120}((\ln P) - (\ln(120-P)))\right] = [.004t + C] \cdot 120$$

$$\ln P - \ln(120-P) = .48t + C$$

Multiply by -1

$$\ln(120-P) - \ln P = -.48t - C$$

$$\ln\left(\frac{120-P}{P}\right) = -.48t - C$$

(4) Solve for P

$$e^{\ln\left(\frac{120-P}{P}\right)} = e^{-.48t-C}$$

$$\frac{120-P}{P} = e^{-.48t}e^{-C}$$

$$\frac{120}{P} - \frac{P}{P} = e^{-.48t}e^{-C}$$

$$\frac{120}{P} - 1 = e^{-.48t}e^{-C}$$

$$\frac{120}{P} = 1 + e^{-.48t}e^{-C}$$

$$\frac{120}{P} = \frac{1 + e^{-.48t}e^{-C}}{1}$$

take the reciprocal of both sides

$$\frac{P}{120} = \frac{1}{1 + e^{-.48t}e^{-C}}$$

$$P = \frac{120}{1 + e^{-.48t}e^{-C}}$$