February 11

SWBAT:

Determine if a series converges using the ratio test

Use the ratio test to determine if $\sum_{n=1}^{\infty} \frac{n^2-1}{2^n}$ converges

$a_n = \frac{n^2-1}{2^n}$

$a_{n+1} = \frac{(n+1)^2-1}{2^{n+1}}$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2-1}{2^{n+1}}}{\frac{n^2-1}{2^n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^2-1}{2^{n+1}} \cdot \frac{2^n}{n^2-1} \right|$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^2-1}{n^2-1} \cdot \frac{2^n}{2^{n+1}} \right| = \lim_{n\to\infty} \left| \frac{n^2-2n+1-1}{n^2-1} \cdot \frac{1}{2} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n^2}{n^2} \cdot \frac{1}{2} \right| = 1 \cdot \frac{1}{2} = \frac{1}{2}$$

$\sum_{n=1}^{\infty} \frac{n^2-1}{2^n}$ converges by the ratio test

Determine if $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$ converges

$a_n = \frac{(n!)^3}{(3n)!}$ $\quad a_{n+1} = \frac{((n+1)!)^3}{(3(n+1))!}$

$$\lim_{n\to\infty}\left|\frac{\frac{((n+1)!)^3}{(3(n+1))!}}{\frac{(n!)^3}{(3n)!}}\right| = \lim_{n\to\infty}\left|\frac{((n+1)!)^3}{(3(n+1))!}\cdot\frac{(3n)!}{(n!)^3}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(3n)!}{(3n+3)!}\cdot\frac{((n+1)!)^3}{(n!)^3}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(3n)(3n-1)\cdots(2)(1)}{(3n+3)(3n+2)(3n+1)(3n)\cdots(2)(1)}\cdot\frac{(n+1)!\,(n+1)!\,(n+1)!}{(n!)(n!)(n!)}\right|$$

$$= \lim_{n\to\infty}\left|\frac{1}{(3n+3)(3n+2)(3n+1)}\cdot\frac{(n+1)(n+1)(n+1)}{1}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(n+1)(n+1)(n+1)}{3(n+1)(3n+2)(3n+1)}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^2}{3(3n+2)(3n+1)}\right| = \lim_{n\to\infty}\left|\frac{(n)^2}{3(3n)(3n)}\right|$$

$$= \lim_{n\to\infty}\left|\frac{n^2}{27n^2}\right| = \frac{1}{27}$$

$\sum_{n=1}^{\infty}\frac{(n!)^3}{(3n)!}$ converges by the Ratio test

Assignment 6

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#1-20 skip multiples of 3