February 12

SWBAT:

Determine if a series converges using the direct comparison test

Direct Comparison Test

Let $\sum a_n$ be a series with no negative terms and $a_n \leq c_n$

If $\sum c_n$ converges, then $\sum a_n$ converges

Let $d_n \leq a_n$

If $\sum d_n$ diverges, then $\sum a_n$ diverges

Direct Comparison Test

Essential Learning Goals:

- Apply properties of Alternating Series, including the error bound
- Use the Ratio Test to determine convergence or divergence
- Use the Comparison Test to determine convergence or divergence
- Find the Lagrange Error Bound for a Taylor Polynomial
- Find the Interval and Radius of Convergence for a Power Series

Use the direct comparison test to determine if $\sum_{n=0}^{\infty} 2^{-n!}$ converges

$a_n = 2^{-n!} = \frac{1}{2^{n!}}$

compare to $c_n = \frac{1}{2^n} = \left(\frac{1}{2}\right)^n$

$\frac{1}{2^{n!}} \leq \frac{1}{2^n} = \left(\frac{1}{2}\right)^n$

$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \rightarrow$ geometric series

$r = \frac{1}{2} < 1$

so, $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$ converges

$\therefore \sum_{n=0}^{\infty} \frac{1}{2^{n!}}$ converges by DCT

Determine if

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{(n!)^2}$$

Converges

$$a_n = \frac{x^{2n}}{(n!)^2}$$

Compare to $c_n = \frac{x^{2n}}{n!}$

$$\frac{x^{2n}}{(n!)^2} \leq \frac{x^{2n}}{n!}$$

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{n!} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots$$

$$= 1 + (x^2) + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \frac{(x^2)^4}{4!} + \cdots = e^{x^2}$$

Converges to e^{x^2}

$\therefore \sum_{n=0}^{\infty} \frac{x^{2n}}{(n!)^2}$ converges by DCT