February 13

SWBAT:

Determine if an Alternating Series Converges

Alternating Series Test

An alternating series $\sum (-1)^n a_n$ converges if:

1. each a_n is positive
2. $a_n \geq a_{n+1}$
3. $\lim_{n \to \infty} a_n = 0$

(Leibniz Test)

Alternating Series Test

Determine if $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{3^n}$ converges

Alternating Series

① $a_n \geq 0$

$\frac{n!}{3^n} > 0$ - Yes

② $a_n \geq a_{n+1}$

$\frac{n!}{3^n} > \frac{(n+1)!}{3^{n+1}} \longrightarrow$ NO

$\frac{n!}{3^n} \leq \frac{(n+1)n!}{3 \cdot 3^n} = \frac{n+1}{3} \cdot \frac{n!}{3^n}$

(for $n \geq 2$

greater than 1

$\sum_{n=1}^{\infty} (-1) \frac{n!}{3^n}$ Diverges by AST

Determine if $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \cdot \ln(n)}$ converges

Alternating Series

① $a_n > 0$

$\frac{1}{n \cdot \ln(n)} > 0 \longrightarrow$ Yes

② $a_n > a_{n+1}$

$\frac{1}{n \cdot \ln(n)} > \frac{1}{(n+1)\ln(n+1)} \longrightarrow$ Yes

③ $\lim_{n \to \infty} a_n = 0$

$\lim_{n \to \infty} \frac{1}{n \cdot \ln(n)} = 0 \longrightarrow$ Yes

$\therefore \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \cdot \ln(n)}$ converges by AST

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

Harmonic Series

Diverges

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$$

Alternating Series

① $\frac{1}{n} > 0 \to$ Yes

② $\frac{1}{n} > \frac{1}{n+1} \to$ Yes

③ $\lim_{n\to\infty} \frac{1}{n} = 0 \to$ Yes

$\therefore \sum_{n=1}^{\infty} (-1)\frac{1}{n}$ converges by AST

"conditionally convergent"