February 25

SWBAT:

Find the interval and radius of convergence

**Radius of Convergence**

Distance from the center of the interval of convergence to the "edge"

The length of convergence divided by 2

IoC: $a < x < b$

radius: $\left| \frac{b-a}{2} \right|$

Find the radius of convergence

$$\sum_{n=0}^{\infty}(x-3)^n$$

Geometric

$r=x-3 \rightarrow |x-3|<1$

$-1<x-3<1 \rightarrow$ IoC: $2<x<4$

RoC: $\frac{4-2}{2}=1$

$$\sum_{n=0}^{\infty}\frac{(x+4)^n}{5^n}$$

geometric

$\sum_{n=0}^{\infty}\left(\frac{x+4}{5}\right)^n$ $r=\frac{x+4}{5}$

converges when $\left|\frac{x+4}{5}\right|<1$

$-1<\frac{x+4}{5}<1$

$-5<x+4<5$

IoC: $-9<x<1$

RoC: $\frac{1-(-9)}{2}=5$

$$\sum_{n=0}^{\infty}\left(\frac{x^2-4}{6}\right)^n$$

Geometric

$r=\frac{x^2-4}{6}$

$\left|\frac{x^2-4}{6}\right|<1$ $-6<x^2-4<6$

$-1<\frac{x^2-4}{6}<1$ $-2<\underbrace{x^2<10}$

$\sqrt{x^2}<\sqrt{10}$

$x<\pm\sqrt{10}$

$-\sqrt{10}<x<\sqrt{10}$

IoC

RoC: $\frac{\sqrt{10}-(-\sqrt{10})}{2}=\sqrt{10}$

Use the Ratio test to find the radius of convergence

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n}}{2n}$$

$a_n=(-1)^{n+1}\frac{x^{2n}}{2n}$ $a_{n+1}=(-1)^{n+1+1}\frac{x^{2(n+1)}}{2(n+1)}$

$$\lim_{n\to\infty}\left|\frac{\frac{x^{2n+2}}{2n+2}}{\frac{x^{2n}}{2n}}\right|=\lim_{n\to\infty}\left|\frac{x^{2n+2}}{2n+2}\cdot\frac{2n}{x^{2n}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{x^{2n+2}}{x^{2n}}\cdot\frac{2n}{2n+2}\right|=\lim_{n\to\infty}\left|x^2\cdot\frac{2n}{2n+2}\right|=|x^2|$$

$$\lim_{n\to\infty}\left|\frac{x^{2n}x^2}{x^{2n}}\cdot\frac{2n}{2n+2}\right|$$

Converges when $|x^2|<1$

$-1<x^2<1$

IoC: $-1<x<1$

RoC: $\frac{1-(-1)}{2}=1$

What is the interval of convergence?

Check endpoints

$x=1$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1^{2n}}{2n}=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n}$$

$$=\frac{1}{2}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} \leftarrow$$ Alternating Harmonic Series $\rightarrow$ converges

$-1\leq x\leq 1$

$x=-1$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(-1)^{2n}}{2n}$$

$$=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n}$$

converges

Use the Ratio test to find the radius of convergence

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{3n}$$

$a_n = \frac{(x-2)^n}{3n}$ $\quad a_{n+1} = \frac{(x-2)^{n+1}}{3(n+1)}$

$$\lim_{n\to\infty}\left|\frac{\frac{(x-2)^{n+1}}{3(n+1)}}{\frac{(x-2)^n}{3n}}\right| = \lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{3(n+1)}\cdot\frac{3n}{(x-2)^n}\right| = |x-2|$$

Converges when $|x-2|<1$

RoC = 1 $\quad -1 < x-2 < 1$

$1 < x < 3$

What is the interval of convergence?

Check endpoints

$x=1$

$$\sum_{n=1}^{\infty}\frac{(1-2)^n}{3n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{3n}$$

Alternating Harmonic Series $\rightarrow$ converges at $x=1$

$x=3$

$$\sum_{n=1}^{\infty}\frac{(3-2)^n}{3n} = \sum_{n=1}^{\infty}\frac{1^n}{3n} = \sum_{n=1}^{\infty}\frac{1}{3n}$$

Harmonic Series $\rightarrow$ Diverges at $x=3$

IoC: $1 \leq x < 3$

$$\sum_{n=0}^{\infty}\frac{nx^n}{n+2}$$

find the IoC