February 26

SWBAT:

Find the error of a Taylor Polynomial using the Lagrange Error Bound

Taylor Polynomial

$$P(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \cdots + \frac{f^{(n)}(0)x^n}{n!}$$

$$f(x) = P(x) + \underbrace{R(x)}_{\text{error}}$$

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

Lagrange Error Bound

"Remainder in a Taylor Polynomial"

maximum value for the n+1 derivative (for some z between a and x)

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$$

centered at a

What is the Maximum Error?

$$|R_n(x)| \leq M \frac{(x-a)^{n+1}}{(n+1)!}$$

Constant

(value of n^{th}+1 derivative at z)

(max value of n^{th}+1 derivative)

Determine how closely the 4th degree Taylor polynomial for cos(x) approximates the true value on the interval (-0.5, 0.5)

$$\cos\left(\frac{1}{4}\right) - \left(1 - \frac{\left(\frac{1}{4}\right)^2}{2!} + \frac{\left(\frac{1}{4}\right)^4}{4!}\right) = \text{Error}$$

$n=4$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$f^{(4)}(x) = \cos x$

$f^{(5)}(x) = -\sin x$

$$|R_4(x)| \leq \frac{f^{(5)}(z)}{(5)!}(x-0)^5$$

$$|R_4(x)| \leq \frac{1}{5!}x^5$$

at $x = \frac{1}{4}$

$$\frac{1}{5!}\left(\frac{1}{4}\right)^5$$

Max Error

$$\frac{1}{5!}(.5)^5$$