February 27

SWBAT:

Find the error in an alternating series and using the Lagrange Error Bound

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Determine how closely the $\boxed{3^{rd}}$ degree Taylor polynomial for e^x approximates the true value on the interval $\underbrace{(1.8, 2.2)}_{\text{centered at } x=2}$

n

$n = 3$

$$|R_n(x)| \le \frac{f^{(n+1)}(z)}{(n+1)!}(x-2)^{n+1}$$

$$|R_3(x)| \le \frac{f^4(z)}{4!}(x-2)^4 \qquad f^{(4)}(x) = e^x$$

$$|R_3(x)| \le \frac{e^{2.2}}{4!}(x-2)^4 = \frac{e^{2.2}}{4!}(2.2-2)^4$$

$$|R_3(x)| \le 6.0167 \times 10^{-4}$$

Let $f(x)=\sin\left(5x+\frac{\pi}{4}\right)$ and let P(x) be the $\underbrace{\text{third-degree}}_{n=3}$ Taylor Polynomial for f about x = 0. Use the Lagrange error bound to show that $\underbrace{\left|f\left(\frac{1}{10}\right)-P\left(\frac{1}{10}\right)\right|}_{\text{error}}<\frac{1}{100}$

$x=\frac{1}{10}$

$$|R_3(x)|\le\frac{f^4(z)}{4!}\left(\frac{1}{10}-0\right)^4$$

$$|R_3(x)|\le\frac{5^4}{4!}\left(\frac{1}{10}\right)^4<\frac{1}{100}$$

$$|R_3(x)|\le\frac{5^4}{4!}\left(\frac{1}{10000}\right)<\frac{1}{100}$$

$f(x)=\sin\left(5x+\frac{\pi}{4}\right)$

$f'(x)=5\cos\left(5x+\frac{\pi}{4}\right)$

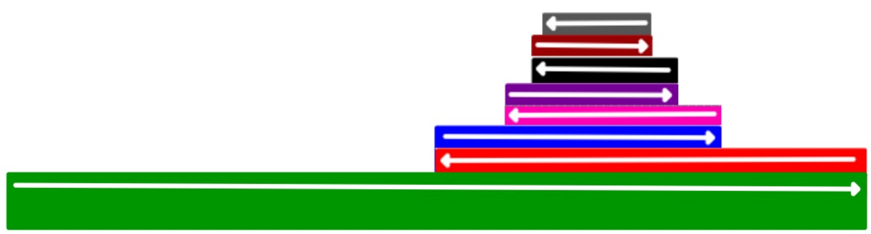
$f''(x)=-5^2\sin\left(5x+\frac{\pi}{4}\right)$

$f'''(x)=-5^3\cos\left(5x+\frac{\pi}{4}\right)$

$f^4(x)=5^4\sin\left(5x+\frac{\pi}{4}\right)$

max value $=5^4$

Alternating Harmonic Series



$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}$$

Error $\le$ |next term| $\le\frac{1}{9}$

Error in Alternating Series

$\text{Error} \leq |a_{n+1}|$

The error for the n^{th} partial sum is less than the $n^{th}+1$ term

$$\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{1}{n+2}\right)$$

Find the error if the 50^{th} partial sum is used to approximate the sum

$n = 50$

$n+1 = 51$

$$\text{error} \leq \left|(-1)^{52}\frac{1}{51+2}\right| = \frac{1}{53}$$