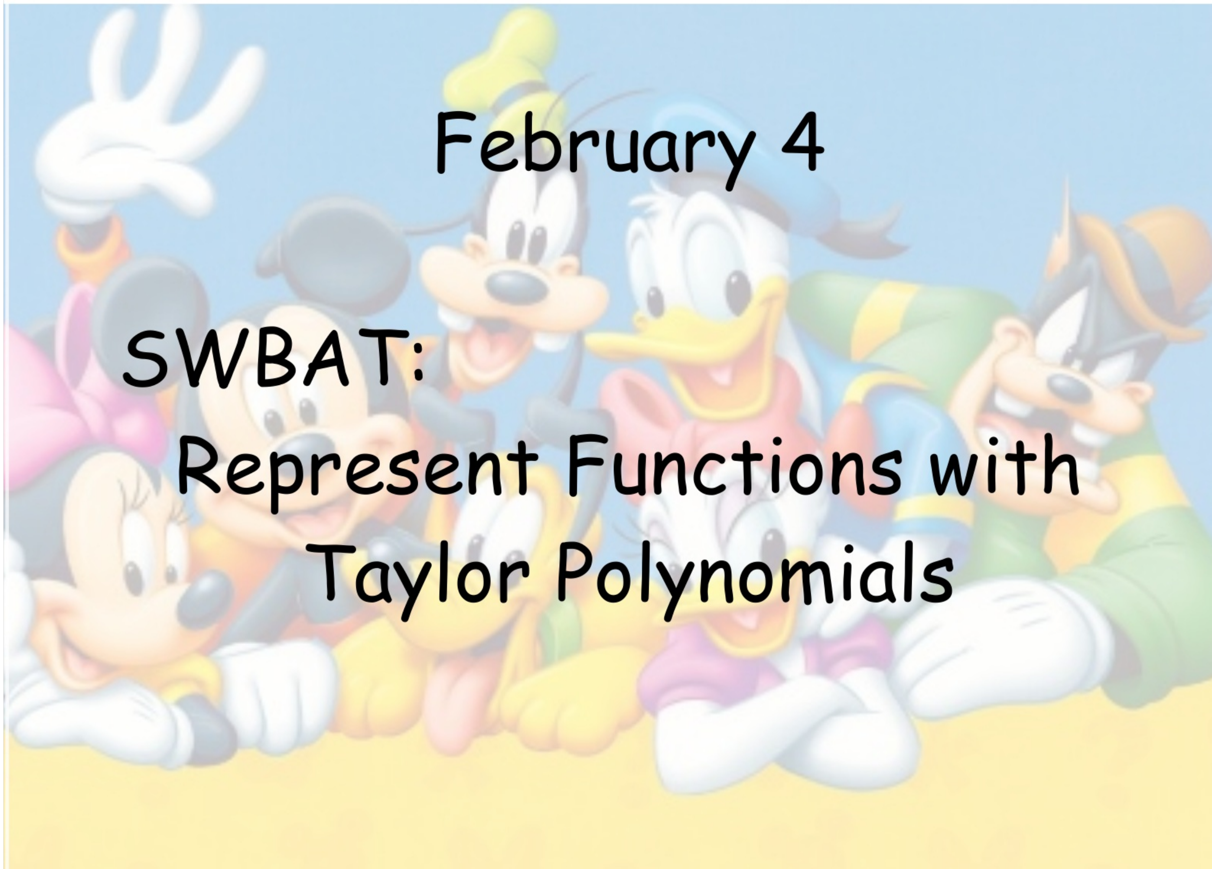

$$\frac{\sin x}{1-x} = \sin x\left(\frac{1}{1-x}\right)$$

#21

$$= \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right)\left(1 + x + x^2 + x^3 + \cdots\right)$$

$$= x + x^2 + 1x^3 + x^4 - \frac{x^3}{6} - \frac{x^4}{6} \qquad \frac{6}{6}x^3 - \frac{x^3}{6}$$

$$= x + x^2 + \frac{5x^3}{6} + \frac{5x^4}{6}$$

#13

$$e^{x-2} = e^x(e^{-2})$$

$$= e^{-2}(e^x) = e^{-2}\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots\right)$$

$$= e^{-2} + e^{-2}x + \frac{e^{-2}x^2}{2} + \cdots + \frac{e^{-2}x^n}{n!} + \cdots$$

IoC: $-\infty < x < \infty$

Construct a 4th degree Polynomial where
$P(0)=1$
$P'(0)=2$
$P''(0)=3$
$P'''(0)=4$
$P^{(iv)}(0)=5$

$P(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4$

$1 = c_0 + c_1(0) + c_2(0)^2 + c_3(0)^3 + c_4(0)^4$ $\qquad c_0 = 1$

$P'(x) = c_1 + 2c_2x + 3c_3x^2 + 4c_4x^3$

$2 = c_1 + 2c_2(0) + 3c_3(0)^2 + 4c_4(0)^3$ $\qquad c_1 = 2$

$P''(x) = 2c_2 + 6c_3x + 12c_4x^2$

$3 = 2c_2 + 6c_3(0) + 12c_4(0)^2$ $\qquad c_2 = \frac{3}{2}$

$P'''(x) = 6c_3 + 24c_4x$

$4 = 6c_3 + 24c_4(0)$ $\qquad c_3 = \frac{4}{6} = \frac{2}{3}$

$P^{(iv)}(x) = 24c_4$

$5 = 24c_4$ $\qquad c_4 = \frac{5}{24}$

$$P(x) = 1 + 2x + \frac{3}{2}x^2 + \frac{4}{6}x^3 + \frac{5}{24}x^4$$

$$P(x) = 1 + 2x + \frac{3}{2}x^2 + \frac{4}{6}x^3 + \frac{5}{24}x^4$$

$$\frac{P(0)}{0!} \quad \frac{P'(0)}{1!} \quad \frac{P''(0)}{2!} \quad \frac{P'''(0)}{3!} \quad \frac{P^{iv}(0)}{4!}$$

Taylor Series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \cdots + \frac{f^{(n)}(0)x^n}{n!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(0)x^n}{n!}$$

centered at zero (Maclaurin Series)

Taylor Polynomial

finite number of terms

$$\sum_{n=0}^{K} \frac{f^{(n)}(0)\,x^n}{n!}$$

K = order/degree of polynomial

find the 4^{th} degree polynomial that approximates $f(x)=\sqrt{1+2x}$ around $x=0$

$f(0)=\sqrt{1+2(0)}=1$

$f(x)=(1+2x)^{1/2}$

$f'(x)=\frac{1}{2}(1+2x)^{-1/2}(2)$

$=(1+2x)^{-1/2}$ $\quad f'(0)=1$

$f''(x)=-\frac{1}{2}(1+2x)^{-3/2}(2)$

$=-(1+2x)^{-3/2}$ $\quad f''(0)=-1$

$f'''(x)=+\frac{3}{2}(1+2x)^{-5/2}(2)$

$=3(1+2x)^{-5/2}$ $\quad f'''(0)=3$

$f^{IV}(x)=-\frac{15}{2}(1+2x)^{-7/2}(2)$

$=-15(1+2x)^{-7/2}$ $\quad f^{IV}(0)=-15$

$$P(x)=1+x+\frac{-x^2}{2!}+\frac{3x^3}{3!}+\frac{-15x^4}{4!}$$