February 5

SWBAT:
Represent Functions with Taylor Polynomials

$$e^{\sin x} = e^{x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$= 1 + \left(x - \frac{x^3}{3!} + \boxed{\frac{x^5}{5!}}\right) + \frac{\left(x - \frac{x^3}{3!} + \boxed{\frac{x^5}{5!} + \cdots}\right)^2}{2!}$$

$$+ \frac{\left(x - \boxed{\frac{x^3}{3!}} + \boxed{\frac{x^5}{5!}}\right)^3}{3!} + \frac{\left(x - \boxed{\frac{x^3}{3!}} + \boxed{\frac{x^5}{5!}}\right)^4}{4!}$$

$$= 1 + x - \frac{x^3}{3!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{2x^4}{2!3!}$$

$$= 1 + x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^4}{3!}$$

$$\left(x - \frac{x^3}{3!}\right)\left(x - \frac{x^3}{3!}\right)$$

$$x^2 - \frac{x^4}{3!} + \frac{x^6}{3!(3!)} - \frac{x^4}{3!}$$

$$\frac{x^2 - \frac{2x^4}{3!}}{2!}$$

$$\left(x - \frac{x^3}{3!}\right)\left(x - \frac{x^3}{3!}\right)\left(x - \frac{x^3}{3!}\right)$$

$$\left(x^2 - \frac{x^4}{3!} + \frac{x^6}{3!(3!)} - \frac{x^4}{3!}\right)\left(x - \frac{x^3}{3!}\right)$$

$$\left(x^2 - \frac{2x^4}{3!}\right)\left(x - \frac{x^3}{3!}\right)$$

$$x^3 + \frac{-x^5}{3!} - \frac{2x^5}{3!} + \frac{2x^7}{(3!)^2}$$

Taylor Polynomial Centered at x = 0

$$P(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^n(0)}{n!}x^n$$

Series

$$f(x) \approx f(0) + f'(0)x + \cdots + \frac{f^n(0)}{n!}x^n + \cdots$$

Taylor Polynomial Centered at x = a

$$P(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^n(a)}{n!}(x-a)^n$$

Series

$$f(x) \approx f(a) + f'(a)(x-a) + \cdots + \frac{f^n(a)}{n!}(x-a)^n + \cdots$$

find the 3rd degree Taylor polynomial approximation of e^{-x} centered at $x=1$

$f(x) = e^{-x}$

$f(1) = e^{-1} = \frac{1}{e}$

$f'(x) = -e^{-x}$

$f'(1) = -\frac{1}{e}$

$f''(x) = e^{-x}$

$f''(1) = \frac{1}{e}$

$$P_3(x) = \frac{1}{e} + \frac{-1}{e}(x-1) + \frac{\frac{1}{e}(x-1)^2}{2!} + \frac{\frac{-1}{e}(x-1)^3}{3!}$$

$f'''(x) = -e^{-x}$

$f'''(1) = -\frac{1}{e}$

approximate $f(1.1) \approx P_3(1.1)$

$$= \frac{1}{e} - \frac{1}{e}(1.1-1) + \frac{1}{2e}(1.1-1)^2 - \frac{1}{6e}(1.1-1)^3$$

$$f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots + \frac{x^n}{(n+1)!} + \cdots$$

f(x) is given by the Maclaurin Series above (centered at zero)

find $f'(0)$

$$f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots + \frac{x^n}{(n+1)!} + \cdots$$

General Taylor series

$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^n(0)}{n!}x^n$$

$$\frac{f'(0)x}{x} = \frac{x}{2!} = \frac{x\left(\frac{1}{2}\right)}{x}$$

$$f'(0) = \frac{1}{2}$$

$$\frac{\frac{f^{10}(0)}{10!}x^{10}}{x^{10}} = \frac{\frac{x^{10}}{11!}}{x^{10}}$$

$$\frac{f^{10}(0)}{10!} = \frac{1}{11!}$$

$$f^{10}(0) = \frac{1}{11!} \cdot 10!$$

$$= \frac{1}{11}$$