What we have seen so far...

A series is the sum of the terms in a sequence

Test for Divergence:

If the limit of the terms in the sequence is not zero...the series diverges

Some special series:

Geometric Series

$$\sum a(r)^n$$

converges if $|r| < 1$
diverges if $|r| \geq 1$

converges to:

$$\frac{a}{1 - r}$$

P-series

$$\sum \frac{1}{n^p}$$

converges if $p > 1$
diverges if $p \leq 1$

harmonic series:
$p = 1$ --> diverges

Using the sum formula, we can convert between some functions and power series

$$\sum a(r)^n \qquad \text{sum} = \frac{a}{1 - r}$$

$$f(x) = \frac{1}{1 - x} = 1 + x + x^2 + x^3 + \cdots$$

Some functions it is good to know the series expansion for:

IoC: all x-values

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n\left(\frac{x^{2n+1}}{(2n+1)!}\right) + \cdots = \sum_{n=0}^{\infty}(-1)^n\left(\frac{x^{2n+1}}{(2n+1)!}\right)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n\left(\frac{x^{2n}}{(2n)!}\right) + \cdots = \sum_{n=0}^{\infty}(-1)^n\left(\frac{x^{2n}}{(2n)!}\right)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

IoC: $-1 < x < 1$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots = \sum_{n=0}^{\infty}(-1)^n x^n$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$$

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^n\frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$$

You can take the derivative or integral of any power series to find the series for a different function.

When you do this, the interval of convergence does not change.

The basic Taylor expansion is helpful too!

(centered at x = 0
and at x = a)

$$f(x) \approx \sum_{n=0}^{\infty} \frac{f^{n}(0)}{n!} x^{n}$$

$$\sum_{n=0}^{\infty} \frac{f^{n}(a)}{n!} (x-a)^{n}$$

We still need to learn:

More tests to see if any series will converge or diverge

How to find the error

How to find the radius of convergence