February 8

SWBAT:

Determine if a series converges using the ratio test

6. Let $f(x) = \sin\left(x^2\right) + \cos x$. The graph of $y = \left|f^{(5)}(x)\right|$ is shown above.

(a) Write the first four nonzero terms of the Taylor series for $\sin x$ about $x = 0$, and write the first four nonzero terms of the Taylor series for $\sin\left(x^2\right)$ about $x = 0$.

(b) Write the first four nonzero terms of the Taylor series for $\cos x$ about $x = 0$. Use this series and the series for $\sin\left(x^2\right)$, found in part (a), to write the first four nonzero terms of the Taylor series for f about $x = 0$.

(c) Find the value of $f^{(6)}(0)$.

(d) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = \left|f^{(5)}(x)\right|$ shown above, show that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

$$x^2 - \frac{x^2}{2!} = x^2 - \frac{1}{2}x^2 = \frac{1}{2}x^2$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$\sin x^2 = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$f(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \left(-\frac{1}{3!} - \frac{1}{6!}\right)x^6$$

$$\frac{f^{(6)}(0)}{6!}x^6 = \left(-\frac{1}{3!} - \frac{1}{6!}\right)x^6$$

Simplify:

$n = 10$

$$\frac{3(n+1)!\,x^4}{5(n)!\,x^5} = \frac{3\overset{11}{(n+1)}\cancel{\overset{10}{(n)}}\cancel{\overset{9}{(n-1)}}\cancel{\overset{8}{(n-2)}}\cdots\cancel{\overset{2}{(2)}}\cancel{\overset{1}{(1)}}\,x^4}{5\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{(2)}\cancel{(1)}\,x^5} = \frac{3(n+1)}{5x}$$

$$\frac{2^{n+2}(n)!}{2^n(n+2)!} = \frac{\cancel{2^n}2^2\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{(2)}\cancel{(1)}}{\cancel{2^n}(n+2)(n+1)\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{(2)}\cancel{(1)}} = \frac{2^2}{(n+2)(n+1)}$$

$$\frac{(n+1)\,x^{n+1}\,7^n}{7^{n+1}(n)\,x^n} = \frac{(n+1)\,\cancel{x^n}\,x\,\cancel{7^n}}{\cancel{7^n}(7)(n)\,\cancel{x^n}} = \frac{(n+1)x}{7n}$$

$\dfrac{3(n+1)!\,x^4}{5(n)!\,x^5}$	$\dfrac{2^2}{(n+2)(n+1)}$
$\dfrac{2^{n+2}(n)!}{2^n(n+2)!}$	$\dfrac{(n+1)\,x}{7(n)}$
$\dfrac{(n+1)\,x^{n+1}\,7^n}{7^{n+1}(n)\,x^n}$	$\dfrac{3(n+1)}{5x}$

$$\sum_{n=1}^{\infty} a_n \text{ is a series}$$

$$\text{and } \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L \quad (\rho)$$

If $L > 1$, the series diverges

If $L < 1$, the series converges

If $L = 1$, the test fails

Ratio Test

Determine if $\sum_{n=1}^{\infty} \frac{4^n}{5^n+1}$ converges

$$a_n = \frac{4^n}{5^n+1} \qquad a_{n+1} = \frac{4^{n+1}}{5^{n+1}+1}$$

$$\lim_{n\to\infty}\left|\frac{\frac{4^{n+1}}{5^{n+1}+1}}{\frac{4^n}{5^n+1}}\right| = \lim_{n\to\infty}\left|\frac{4^{n+1}}{5^{n+1}+1}\cdot\frac{5^n+1}{4^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{4^{n+1}}{4^n}\cdot\frac{5^n+1}{5^{n+1}+1}\right|$$

$$= \lim_{n\to\infty}\left|4\cdot\frac{5^n+1}{5^{n+1}+1}\right| = \lim_{n\to\infty}\left|4\cdot\frac{5^n}{5^{n+1}}\right| =$$

$$= \lim_{n\to\infty}\left|4\cdot\frac{1}{5}\right| = \frac{4}{5}$$

$\sum_{n=1}^{\infty} \frac{4^n}{5^n+1}$ converges by the Ratio test