# January 11

SWBAT:

Find the sum of a geometric series

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Geometric Series

Sum of the terms in a geometric sequence

$$\sum_{n=1}^{\infty} b(r)^{n-1}$$

$$= \sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots + ar^{n-1} + \ldots$$

6. Write an expression for the sum of the areas of the infinite number of rectangles formed. What is the value of this sum? Why?

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots = 1$$

7. Express your answer from Question 6 in sigma notation.

$$\sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{1}{2}\right)^{n-1} \qquad \sum_{n=1}^{\infty}1\left(\frac{1}{2}\right)^{n} \qquad \sum_{n=1}^{\infty}\left(\frac{1}{2^{n}}\right)$$

8. Instead of halving the side of the square, suppose that we doubled its size and continued to double a side of each subsequent square formed.
   a. Express the sum of the areas of these squares as an infinite sum.

$$1+2+4+8+\cdots$$

   b. What happens to this sum as the number of squares increases? Explain your answer.

9. Instead of halving the length or width of each of the rectangles, suppose that we multiplied the rectangle's length or width by 1/3, giving us the series $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\ldots$

   Do you think that the sum of the series would be finite or infinite? Explain.

10. Give an example of an infinite geometric series that you think would have a finite sum and an example of one that you think would not have a finite sum. Explain your reasoning.

$$\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n} \qquad \sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^{n} \qquad \sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^{n} \qquad \sum_{n=1}^{\infty}\left(\frac{1}{216}\right)^{n}$$

11. Based on the information above, what do you conjecture must be true about the ratio of the consecutive terms of an infinite geometric series for the series to have a finite sum?

**Move to page 1.3.**

12. For what values of the ratio *r* does an infinite geometric series appear to have a finite sum?



| | |
|---|---|
| When does a geometric series converge? | when $\lvert r\rvert < 1$<br>the series converges to $\frac{a}{1-r}$<br>Sum $=\frac{a}{1-r}$<br>$a=$ starting number (first # in sequence) |
| Interval of convergence | $\lvert r\rvert < 1$<br>$-1 < r < 1$ |
| When does a geometric series diverge? | $\lvert r\rvert \geq 1$ |

$$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)\left(\frac{5}{4}\right)^{n-1}$$

converge or diverge?

diverge

because $r=\frac{5}{4}>1$

$\left(a=\frac{2}{3}\right)$

$$\sum_{n=1}^{\infty}7\left(\frac{5}{7}\right)^{n-1}$$

converge or diverge?

converge

because $r=\frac{5}{7}<1$

converges to $\frac{7}{1-\frac{5}{7}}=\frac{7}{\frac{2}{7}}=\frac{49}{2}$