January 16

SWBAT:

Apply the integral test to determine if a series converges or diverges

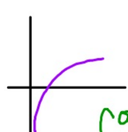
$$\sum_{n=1}^{\infty} \frac{1}{n+3}$$

$$\int_1^{\infty} \frac{1}{x+3}\,dx$$

$$\lim_{R\to\infty} \int_1^{R} \frac{1}{x+3}\,dx = \lim_{R\to\infty} \ln|x+3| \Big|_1^R$$

$$= \lim_{R\to\infty} \ln(R+3) - \ln(1+3)$$

$$= \infty - \ln(4) = \infty$$



$$\int_1^{\infty} \frac{1}{x+3}\,dx \text{ diverges}$$

$$\underbrace{\therefore}_{\text{Therefore}} \sum_{n=1}^{\infty} \frac{1}{x+3} \text{ diverges}$$

$$\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^{3/5}}$$

$$(n^2)^{3/5} = n^{6/5}$$

$$\int_1^{\infty} \frac{x}{(x^2+1)^{3/5}}\,dx$$

$$= \lim_{a\to\infty} \int_1^{a} \frac{x}{(x^2+1)^{3/5}}\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$x=1, u=2$

$x=a, u=a^2+1$

$$\lim_{a\to\infty} \int_2^{a^2+1} \frac{x}{u^{3/5}}\,\frac{du}{2x}$$

$$= \lim_{a\to\infty} \int_2^{a^2+1} \frac{1}{2u^{3/5}}\,du = \lim_{a\to\infty} \frac{1}{2}\int_2^{a^2+1} u^{-3/5}\,du$$

$$= \lim_{a\to\infty} \frac{1}{2}\left(\frac{5}{2}u^{2/5}\Big|_2^{a^2+1}\right)$$

$$= \lim_{a\to\infty} \frac{5}{4}(a^2+1)^{2/5} - \frac{5}{4}(2)^{2/5} = \infty$$

$\int_1^{\infty} \frac{x}{(x^2+1)^{3/5}}\,dx$ diverges

$\therefore \sum_{n=1}^{\infty} \frac{n}{(n^2+1)^{3/5}}$ diverges

$$\sum_{n=1}^{\infty} \frac{1}{3^{\ln n}} = \int_1^{\infty} \frac{1}{3^{\ln x}}\,dx$$

$$3 = e^{\ln 3}$$

$$\frac{1}{3^{\ln x}} = \frac{1}{(e^{\ln 3})^{\ln x}}$$

$$= \frac{1}{e^{\ln x \cdot \ln 3}} = \frac{1}{x^{\ln 3}}$$

$$(e^{\ln x})^{\ln 3}$$