January 22

SWBAT:

Define transcendental functions using polynomial expansions

$$\sin x - x \approx \frac{-x^3}{6}$$

$$\sin x \approx \frac{-x^3}{6} + x = x - \frac{x^3}{6}$$

$$\sin x - \left(x - \frac{x^3}{6}\right) \approx \frac{x^5}{120}$$

$$\sin x \approx \frac{x^5}{120} + \left(x - \frac{x^3}{6}\right)$$

$$\sin x \approx x - \frac{x^3}{6} + \frac{x^5}{120}$$

$$\sin x \approx x - \frac{x^3}{6} + \frac{x^5}{120}$$

$$\approx \frac{x^1}{1} - \frac{x^3}{3\cdot 2\cdot 1} + \frac{x^5}{5\cdot 4\cdot 3\cdot 2\cdot 1} - \frac{x^7}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$$

$$\approx \frac{x^1}{1} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$$

$$\cos x - \left(-\frac{x^2}{2} + 1\right) \approx \frac{x^4}{24}$$

$$\cos x \approx \frac{x^4}{24} - \frac{x^2}{2} + 1 = 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

$$\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad 0! = 1$$

$$\sin x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^{n} \frac{x^{2n}}{(2n)!}$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\sin x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

derivative

$$= 1 - \frac{3x^2}{3\cdot 2\cdot 1} + \frac{5x^4}{5\cdot 4\cdot 3\cdot 2\cdot 1} + \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \cos x$$