January 24

SWBAT:
Express rational functions as a power series

Express the following series in sigma notation

$1 + .1 + .01 + .001 + \cdots$

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$$\sum_{n=0}^{\infty} (.1)^n = \sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n$$

Geometric series converges because $r = \frac{1}{10} < 1$

$$\text{Sum} = \frac{a}{1-r} = \frac{1}{1-\frac{1}{10}} = \frac{1}{\frac{9}{10}} = \frac{10}{9}$$

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Geometric Series:

Geometric series are represented by $\sum_{n=1}^{\infty} a(r)^{n-1}$.

A geometric series converges when $|r| < 1$

The sum of a geometric series is $\frac{a}{1-r}$

Generalize the formula by substituting 1 for ~~b~~ a and x for *r*. Write the first 5 terms of the expansion when *a* = 1 and *r* = *x*.

Eq 2 $\frac{1}{1-x} = \sum_{n=1}^{\infty} 1(x)^{n-1} = 1 + x + x^2 + x^3 + x^4 + \cdots$

What is the **interval of convergence** for $f(x) = \frac{1}{1-x}$?

$|r| < 1$

$|x| < 1$

$-1 < x < 1$

Interval of Convergence

The interval where the polynomial expansion matches the function

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

I.o.C. $-\infty < x < \infty$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

I.o.C $-1 < x < 1$

Rational Functions written as a series of polynomials are called **Power Series**.

Power Series

An expression in the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n + \ldots$$

Is a **power series centered at x = 0**.

An expression in the form

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots + c_n(x-a)^n + \ldots$$

Is a **power series centered at x = a**