January 25

SWBAT:
Express rational functions as a power series

The power series we just developed can be used a template to find power series of other functions.

Given that $\frac{1}{1-x}=1+x+x^2+x^3+...+x^n+...=\sum_{n=0}^{\infty} x^n$, do the following examples.

Example 7. Write a power series that in both expanded form and sigma notation that represents $\frac{1}{1+x}$. What is the interval of convergence?

$|r|<1$

$$\frac{1}{1+x}=\frac{1}{1-(-x)}=1+(-x)+(-x)^2+(-x)^3+\cdots$$

$$=1-x+x^2-x^3+x^4+\cdots+(-x)^n+\cdots$$

$$=1-x+x^2-x^3+\cdots+(-1)^n x^n+\cdots=\sum_{n=0}^{\infty}(-1)^n x^n$$

IoC:

$|-x|<1$

$-1<-x<1$

$-1<x<1$

The power series we just developed can be used a template to find power series of other functions.

Given that $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n$, do the following examples.

Example 8. Write a power series that in both expanded form and sigma notation that represents $\dfrac{x}{1+x}$. What is the interval of convergence?

$$\frac{x}{1+x} = x\left(\frac{1}{1+x}\right) = x\left(\frac{1}{1-(-x)}\right)$$

$$= x\left(1 + (-x) + (-x)^2 + (-x)^3 + \cdots + (-x)^n + \cdots\right)$$

$$= x\left(1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots\right)$$

$$= x - x^2 + x^3 - x^4 + \cdots + (-1)^n x^{n+1} + \cdots = \sum_{n=0}^{\infty} (-1)^n x^{n+1}$$

IoC:

$$|r| < 1$$

$$|-x| < 1$$

$$-1 < -x < 1$$

$$-1 < x < 1$$

The power series we just developed can be used a template to find power series of other functions.

Given that $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n$, do the following examples.

Example 9. Write a power series that in both expanded form and sigma notation that represents $\dfrac{1}{1-3x}$. What is the interval of convergence?

$$\frac{1}{1-3x} = 1 + (3x) + (3x)^2 + (3x)^3 + \cdots + (3x)^n + \cdots$$

$$= 1 + 3x + 3^2x^2 + 3^3x^3 + \cdots + 3^n x^n + \cdots$$

$$= \sum_{n=0}^{\infty} 3^n x^n = \sum_{n=0}^{\infty} (3x)^n$$

IoC:

$$|3x| < 1$$

$$\frac{-1}{3} < \frac{3x}{3} < \frac{1}{3}$$

$$-\frac{1}{3} < x < \frac{1}{3}$$